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# RELATIONS OF THE TEMPERATURE OF GEOTHERMAL ENERGY SYSTEMS

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## I. History background of the investigated problem

Investigation of the heat transfer in boreholes has been the task of the geophysics hydrogeology and the civil engineering for a long time. Determination of the heat loss of the produced thermal water has been used by particular empirical relationships until the 70 (BÉLTELKI, 1971, LIEBE, 1976).

The exact mathematical description of the heat conduction and heat transfer began earlier in the field of physics, chemistry and process engineering. The heat transfer phenomena, was synthetized in the most general level by CARSLAW and JAEGER (1947). Solutions of the differential equation of the heat conduction were elaborated for different geometries, different initial and boundary conditions. These solutions are mainly analytic, but many numerical examples are introduced.

The first analytic solution of the heat transfer in boreholes in the earth science was given by BOLDIZSÁR (1958). The transient differential equation of heat conduction was written for the adjacent rock mass around the well. The equation was transformed to a BESSEL differential equation by LAPLACE transformation. The solution is obtained in the form of BESSEL functions of zeroth and first order. BOLDIZSÁR assumed an infinitely large heat transfer coefficient between the flowing fluid and the borehole wall.

RAMEY (1962), introduced a transient heat conduction coefficient, expanded into series the BESSEL functions. He took into consideration the thermal resistance of the well structure. The well was approximated by a tubing of constant diameter from the bottom to the wellhead and it was characterized by a single overall heat transfer coefficient. RAMEY simplified BOLDIZSÁR's method adapted it for field circumstances. Many followers of RAMEY gave sophisticated solutions for some details of the original method.

WILLHITE (1967) improved RAMEY's method, in which the overall heat transfer coefficient was determined with higher accuracy.

PÁPAY (1985) elaborated a brand new method for investigation of temperature distribution of oil and natural gas wells. He recognized the analogy between the differential equations of flow through porous media and the heat conduction. Moreover the boundary conditions of both cases are analogous. PÁPAY appointed that, the fluid flow toward wells and the heat conduction around the wells can be describe, parabolic partial differential equations. He improved Van EVERDINGEN's (1949) method which originally was elaborated for transient flow toward wells in the reservoir mechanics. Based on this method the borehole heat transfer problem was solved and the thermal skin was introduced too.

There were published a great number of numerical solutions since the 70's. They are based on the methods of the finite differences or the finite elements. Publications of HOWEL, SETH and PERKINS (1972), LIN and WHEELER (1978), RYBACH (1981), WOLEY (1980) were the most important in this field.

Research activity of borehole heat transfer was started at the Petroleum Department of the University of Miskolc under the direction of SZILAS (1965). These applied research works were aimed to everyday practice of the petroleum industry, improved some details of Ramey's method (BOBOK 1987, CODO 1990, HAZIM 1996, BOBOK and TÓTH 2000).

## II. The aim of the research work

The results produced until now can not apply to any arbitrary borehole heat transfer problem without further applied research work. The knowledge of the temperature distribution of thermal wells is necessary for diverse problems of geothermics, drilling, well completion fluid production and injection. The main objective is obviously the determination of heat losses in geothermal wells. The well completion, cementing, heat insulation of wells are needs responsibly calculated data.

The overall heat transfer coefficient and the heat conductivity of the rock was considered uniform in the previous works. The varying well completion along the depth must be considered to design of the heat insulation, since it refers for a given interval of the well. There are special completed wells – for example the double-function production-reinjection wells-for which the description of the heat transfer phenomenon isn't carried out. The terrestrial heat flow influences also the temperature distribution of the rock mass around the well. This effect wasn't considered until now.

The practical importance of the accurate calculation of the heat losses in the well can be recognized easily. If a hot water well of a mass flow rate of 20 kg/s is cooled with 1 $^{o}$C, the thermal power loss is 83,6 kW! The cost of a mathematical simulation is much more cheaper than the experimental work. After well testing the performance of the well can change substantially. To follow of this change is necessary in order to effectiv production.

In accordance to these, the aim of my dissertation is the calculation of the heat losses of the geothermal energy production, to clarify the reasons of these losses, at last but not least the increase of the efficiency of the system. The tool for these task is the balance equation of the internal energy, together with adequate boundary conditions. The obtained differential equations are solved analitically, the calculated results are compared to experimental data.

## III. The brief description of the research work

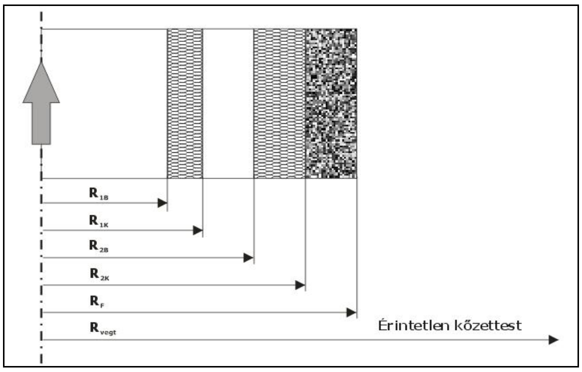
### III.1. The conceptual model of the phenomenon

The temperature of the produced hot water together with its interval energy content decreases substantially from the bottomhole to the wellhead. The reason of this phenomenon is that, the temperature of the upflowing hot water in the tubing is greater then the temperature of the adjacent rock mass around the well. This temperature difference induces a radially outward heat conduction from the well toward the undisturbed, thermally homogeneous rock mass. This heat flow is mostly conductive, but in the high porosity and permeability pleistocene sediments convective heat flow may occure. Due to the heat loss of the thermal water the adjacent rock around the well is warmed up while the inhomogeneity of the temperature field, thus the heat flow is decreased. The temperature of the produced hot water at the wellhead eventually increases after beginning the production, until the whole system – the well and the rock – achieves a steady state.

This unsteady thermal interaction is investigated in the following. The geometry of the system is shown together with its important data.

### III.2. The mathematical model of the phenomenon

The determination of heat losses in the geothermal well is carried out constructing the mathematical model of the phenomenon, solving analytically the obtained differential equations. The three main elements of the mathematical model one the set of basic equations, the obtained boundary conditions together with the necessary material parameters, and the method of the solution. The most important basic equations are the balance equation of the interval energy, together with the conservation law of mass, and the momentum equation.




The whole system is: the flowing geothermal well and its adjacent rock mass. This is separated into two sub-systems. One of this is the flowing thermal water in the tubing, another is the well structure (tubing, casing, cement sheet) and the rock around them. The heat transfer between the flowing water and the tubing wall is forced convection, through the tubing and casing walls it is pure heat conduction, in the fluid-filled annular space between the tubing and the casing it is free convection. Finally, through the cement sheet and in the rock the heat is transferred by conduction. Around the well an axisymmetric heated region develops, in which the terrestrial heat flux and a heat source of varying induced heat flux are superimposed. The balance equations of internal energy are written both of the two sub-systems, thoge lead to a first-order, linear, inhomogeneous differential equation, which can be solved analytically.

This differential equation also valid for heat transfer round telescopic completed well, but the boundary conditions must be adapted for this system. The overall heat transfer coefficient, the transient heat conduction function and heat condutivity of the rock are continouos piecewiseonly-along the depth. Thus the differential equation must be integrated piecewisely, while the boundary conditions are the equal temperatures at the end of a given interval and at the entrame of the next one.

The differential equation remains the same if the well produces from a multilayered aquifer. The different aquifer-layers produces thermal waters of different temperatures. These inhomogeneities are disappears as these waters are mixed satisfying the law of calorimetry. A part-algorhytm is elaborated to get the temperature after the mixing. This common temperature and the corresponding depth as a virtual bottomhole depth is the boundary condition for this problem.

The radius of the heated region around the well is determined by a new method. The originally two-dimensional method of thermal singularities is extended to a three-dimensional axisymmetric case. Its essence is that the unknown function in the differential equations is the heat flux vector instead of the temperature. The second boundary problem of the potential theory, the so-called DIRICHLET-problem is the task must be solued. The obtained differential equation differs one term from the familiar Laplace equation only. This equation is solved analytically. Its solution a transcendent expression, from which the radius of the surface of revolution can be determined by an approximativde numerical procedure only.

There are miscellaneous wells e.g. closed-loop, or double-function producing-injecting wells, in which the balance equation of internal energy must be written for two countercurrent flow in the tubing and the annulus. Based on these, a second-order, linear, homogeneous differential equation is obtained. This equation is solved analitically, the boundary conditions are in accordance with the well completion.

Calculated results are compared in situ experimental data in each cases. This mostly comparison of the computed and the in situ measured wellhead-temperatures, but in some cases the complete temperature distributions were compared both calculated and measured functions along the depth. Measured data are taken partly from the thermal well catalogue of the VITUKI, in other hand from the measurements of the MOL Rt.

## IV. THE NEW SCIENTIFIC RESULTS

**T/1.** The differential equation of the borehole heat transfer leads to the solution

$$T = T_0 + \gamma(z + A) - \gamma A \cdot e^{\frac{(z-H)}{A}}$$

In which

$$A = \frac{mc(k_k + f(t) \cdot R_{1B} U_{1B})}{2\pi R_{1B} U_{1B} k_k}$$

The overall heat transfer coefficient Uvs can be determined by the equation

$$\frac{1}{U_{1B}^*} = \left[\frac{1}{h_{1B}} + \frac{R_{1B}}{k_a}\ln\frac{R_{1K}}{R_{1B}} + \frac{R_{1B}}{R_{1K}}\frac{1}{h_{gy}} + \frac{R_{1B}}{k_a}\ln\frac{R_{2K}}{R_{2B}} + \frac{R_{1B}}{k_{cem}}\ln\frac{R_F}{R_{2K}} + \frac{R_{1B}}{k_k}\ln\frac{R_\infty}{R_F}\right]$$

In this expression the heat transfer coefficient of the fluid-filled annulus ha can be calculated by a try and error procedure only. The last term is the thermal resistance of the heated rock mass around the well. Considering this term the solution becomes more adequate and accurate.

**T/2.** The telescopic completion of the well can be considered by subdividing the whole depth into sections in accordance to the geometry of the easing. The differential equation can be integrated over this intervals separately. These piecewise continuous functions are fitted satisfying the boundary condition that the temperatures at the end points of two joined sections must be equal. Thus the temperature distribution of the i-th vertical section is:

$$T^{(i)} = T_0 + \gamma(z + A_i) + \left[T_{ki}^{(i-1)} - T_0 - \gamma(H_i + A_i)\right]e^{\frac{z-H_i}{A_i}}$$

**T/3**. The temperature difference between the bottomhole and the wellhead of many thermal wells is greater substantially, than it were reasonable because of the heat going away to the adjacent rock around the well. It was proved, that this kind of thermal wells produces from many different aquifer-layer of different temperatures. The mixing of the waters of different temperatures diminishes the temperature, but it is not energy loss, because of mass flow rate is growing, thus the energy content of the flowing water remains the same. This reduction of temperature is significantly greater than the temperature-decrease is caused by the heat loss toward the adjacent rock mass. In the interwal of perforations the heat loss of upflowing water is negligible because of the relatively small temperature difference between the water and the rock. Thus the boundary condition for the temperature distribution of the water can be taken as z = H*, T=T*. here H* is the so-called corrected bottomhole depth, where the mixing of different temperature waters is finished, T* is the common temperature after the mixing, calculated by the equation

$$T_{Ki} = T_0 + \gamma \frac{\sum h_i z_i}{\sum h_i}$$

**T/4.** The known methods elaborated for determination the temperature distribution of geothermal wells neglect the effect of the terrestrial heat flow. A quasi two-dimensional radial heat conduction process is assumed around the well. In accordance of this a cylindrical heated region were developed around the well. The radius of this cylindrical contour surface $R_\infty$ increases monotonically with the time, tending assimptotically to the infinite. Actually, it can be laid down as a fact, that the temprature field is axisymmetric and three-dimensional. The contour of the heated region is a surface of revolution of varying radius along the depth. This radius increases with time until it arrives to a finite steady state value. The steady state contour can be determined by a potential theory method. Its equation

$$\frac{q_\infty r^2}{2} - \frac{C}{8\pi}\ln\frac{r^2 + (z-H)^2}{r^2 + z^2} + \frac{C \cdot r}{4\pi}\left[\operatorname{arctg}\frac{z-H}{r} - \operatorname{arctg}\frac{z}{r}\right] - \frac{1}{8\pi}C \cdot H^2 = 0$$

Subdivided the well into sections, the obtained integral mean of $R_\infty$ can be used to determine the $A_i$ values.

**T/5**. A new mathematical model is elaborated for a doublefunction production-reinjection well. In this single well thermal water is produced through the tubing and the cooled water is reinjected through the annulus to an aquifer of smaller depth. It can be recognized that methods elaborated for countercurrent heat exchangers cannot be applied in this case, because the adjacent rock temperature isn't uniform, but increases with the depth linearly. Applying the balance equation of the internal energy a second order differential equation is obtained, which can be solved analytically. The temperature distribution in the tubing and the annulus is determined by the equations:

$$T_T = B(C_1 x_1 e^{x_1 z} + C_2 x_2 e^{x_2 z}) + (1 + \frac{B}{A}) \cdot (C_1 e^{x_1 z} + C_2 e^{x_2 z}) + T_0 + \gamma z$$

$$T_{Gy} = T_0 + \gamma z + C_1 \cdot e^{x_1 z} + C_2 \cdot e^{x_2 z}$$

**T/6.** A new calculating method is elaborated for the so-called borehole heat exchangers e.g. or closed-loop geothermal well without water production. This method substantially simpler then the welknown procedures. The obtained tempetature distributions in the tubing and the annulus:

$$T_T = B\left(\gamma + x_1 C_1 e^{x_1 z} + x_2 C_2 e^{x_2 z}\right) + \left(1 + \frac{B}{A}\right)\left(T_0 + \gamma z + C_1 e^{x_1 z} + C_2 e^{x_2 z}\right) - \frac{B}{A}\left(T_0 + \gamma z\right)$$

$$T_{Gy} = T_0 + \gamma z + C_1 e^{x_1 z} + C_2 e^{x_2 z}$$

It can be recognized, that in Hungary the temperature increase of the injected water is relatively low, because of the low average heat conductivity of the upper Pannonian sediments. This system may be economic by the heat insulation of the tubing, and to build in a heat pump.

## V. THE APPLICABILITY OF THE OBTAINED RESULTS

It is a wellknown fact, that data of the well register are data of the well testing, measured immediately after the well completion. These data are changing as time goes by the evolution of the heated region around the well is a transient phenomenon. As it tends to a steady state, the temperature of the outflowing water increases sone contigrades relative to the well-test value. In the other hand, later the flow rate of the well, together with the wellhead-temperature decreases because of the pressure drop of the depleting reservoir, or the scaling.
This change can be prognostised adequately by the introduced mathematical modeling, with good accuracy within limits of the temperature measurements. Thus the temperature distribution, the thermal power of the well can be determined at any period of its lifetime, any change of the well completion (eg. Subsequent heat insulation, or fuilt- in a submersible pump.

The accurate determination of the overall heat transfer coefficient makes possible the reliable design of the heat insulation of the well. The insulation of the upper part of the annulus may increase the thermal substantially. Evry single contigrade tempetrature-increment will increase with 84 kW the thermal power of a common geothermal well if the mass-flow rate is 20 kg/s. This procedure, originally elaborated for geothermal wells, can be applied any substantial modification for oil wells, for example the heat insulation of the Szolnok É-I. well of the MOL Rt.

The introduced method is suitable to determine not only the temperature distribution of the flowing fluid in the tubing, but also the temperature of any element of the well: either the tubing or the casing at any arbitrary depth. This calculation can be carried out before the drilling of the well, with knowledge of the future well completion and the geothermal data of the site. Thus reliable temperature data are available for the casing setting.

Most Hungarian geothermal wells produce hot water from multilayered aquifers. The introduced mathematical model is suitable to determine the number of the tapped aquifer-layers to achieve the best efficient performance of the well. It is noteworthy that the optimal temperature and the optimal thermal power occurs at different flow rates. The given way of utilization deetermines that the tempetrature maximum or the thermal power maximum is the best performance point of the well.

The elaborated calculation method is applicable directly the design of borehole heat exchangers or double function production-reinjection wells.

To design of geothermal energy production wells it is essential condition the knowledge a system of reliable design parameters. Thermal conductivities measured on core samples are random values. The introduced mathematicasl model is suitable to determine in situ heat conductivity of a given formation crossed by the borehole. The overall heat transfer coefficient is influenced by the shape and size of the heated region around the well. The shape of the heated region, that is the transient heat conduction function f(t) can be determined using data of performing geothermal well. Thus the design of geothermal wells and prognostise their performance becomes more reliable.

## VI. List of Related Publications


1. BOBOK E.- A. TÓTH SZTERMEN: Temperature distribution in a double-founction production-reinjection geothermal well. Geothermal Resource Council Transactions San Francisco, USA. 2000. Vol.24, pp. 555-559.
2. A. TÓTH SZTERMEN: Heat Transfer in particulary Completed Geothermal Wells Bányászat- Kohászat-Földtan Konferencia Erdélyi Magyar Műszaki Tudományos Társaság Csíksomlyó, Románia, 2001. p. 117.
3. A. TÓTH SZTERMEN: Energy supply in EU countries, MicroCad Konferencia, Miskolc, 2002. pp. 73-81.
4. SZTERMENNÉ TÓTH A.: Termálkút körüli tengelyszimmetrikus hőárammező meghatározása MicroCad Konferencia, Miskolc, 2002. pp. 81-87.
5. SZTERMENNÉ TÓTH A.: Temperature Drop isn't a Pure Heat Loss in Wells Production from Multilayered Aquifers, MicroCad Konferencia, Miskolc, 2002. pp. 87-93.
6. SZTERMENNÉ TÓTH A.: Thermal Losses in Multipurpose geothermal Wells MicroCad Konferencia, Miskolc, 2002. pp. 93-101.
7. SZTERMENNÉ TÓTH A. - RUPERT V.: Megújuló, alternatív energiaforrások felhasználása a hazai energiagazdálkodásban MicroCad KonferenciaMiskolc, 2002. pp. 101-107.
8. A. TÓTH SZTERMEN: Geothermal Resources of Hungary at a Glance, 24th New Zealand Geothermal Workshop, Auckland, New Zealand 2002. pp. 41-45.
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11. A. TÓTH SZTERMEN: Clean Energy for the 21th Century in Hungary, MicroCad Konferencia, Miskolc, 2003. pp. 95-101.
12. BOBOK E.- A. TÓTH SZTERMEN: Geothermal energy from dry holes European Geothermal Conference Szeged, 2003. p. 10.
13. BOBOK E.- A. TÓTH SZTERMEN: Geothermal Energy Production and its Environmental Impact in Hungary Multiple Integrated Uses of Geothermal Resources IGC S12 Reykjavik, Iceland, 2003. pp. 19-25.