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MIKOVINY SÁMUEL DOCTORAL SCHOOL OF EARTH SCIENCES

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Flow Profile Control in Hydraulic Fractured Wells – Foundation of a New Geothermal Energy Production Technology

New scientific achievements of Ph.D. Thesis

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1 INTRODUCTION & TOPIC RELEVANCE

As global energy consumption will increase significantly in the future and the potentially available hydrocarbon resources are finite and become more expensive and difficult to reach, the role of renewable energy resources will be more prominent. It is also promoted by intense environmental endeavors in the form of regulatory and social pressure. Geothermal energy resources have an immense potential, but today do not play a significant role in the renewable energy mix. It is mainly because the economics are highly dependent on the geological and regulatory environment and usually have a higher investment need, thus a longer pay-back time. Without technological innovation, it is unlikely that this energy carrier will play a vital function in the future.

Several different geological formation and technology is available to harvest the energy of the Earth crust. The precursor and inspiration of this research was the Enhanced Geothermal Systems (EGS). There is not a universally accepted definition of EGS, but in most cases it covers technologies that make a geothermal reservoir economically viable to harvest. In most cases these technologies involve one or more injection and production wells to be implemented. Later studies such as Kehrer et al. (2007) and Danko et al. (2018) offers EGS systems where the injection and production are integrated in one well, thus reducing the cost of the total investment. These methods although have some advantages also have some limitations that promotes further investigations in this area.

1.1 Research purpose & research conducted

The main goal of this study is to offer a novel EGS technology and elaborate on the essential theoretical background to facilitate further scientific research and tests in this area. The extensively employed hydraulic fracturing procedure in the oil and gas industry served as the foundation of this method. Although utilization of hydraulic fracturing technology for more efficient geothermal energy production is at the center of several current studies, with this new approach the energy production can be reached in a single well that potentially reduces the investment cost of the project. One of the unique ideas in this method is to create a more efficient flow profile (at the point of heat recovery) in the fracture by forming zones with different permeabilities in the fracture. In this way the surface that is available for heat recovery is not limited by any means and the flow can be optimized in the fracture to reach the maximum heat recovery from the system. It can be reached by a propping agent, the so-called proppant. A conceptual representation of this new approach is illustrated in **Fig. 1-1**, where only one side of the fracture's cross-section is represented.

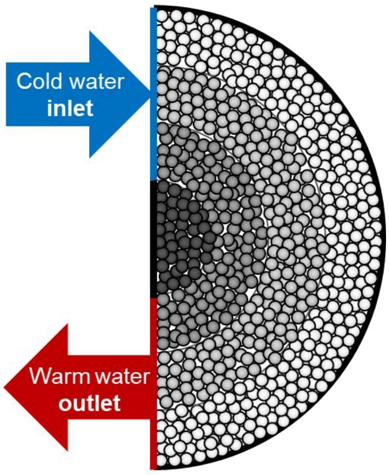

Figure 1-1: Visual representation of the new approach

The different concentric zones have different permeability values, thus providing a flow control mechanism in the fracture. In this thesis, a new semi-analytical approach is developed to reach arbitrary permeability values by mixing two distinct proppant packs together in different mass ratios. With this novel method, it can be determined in what mass percentage should two proppant-pack be mixed to provide the necessary permeability values under reservoir conditions.

During this EGS method, the obtained flow profile in the fracture can be described as a steady-state phenomenon, but in contrast, the heat recovery is constantly decreasing as the fluid draws the reservoir temperature down, so it is a transient phenomenon. This complex problem can be solved only numerically. The most popular approach in computational fluid dynamics is the Finite Element Method (FEM) because it uses the most stable numerical scheme (Rapp, 2023). For this reason, a FEM model was developed in this thesis where the computational domain is initially divided into discrete components so the describing differential equations can be solved numerically. Although the fracture often can be described in 2-dimension, in this special case where the heat recovery mainly takes place perpendicular to the fracture a 3-dimensional model had to be developed. In this research, a state-of-the-art, coupled steady-state – transient simulation was developed where the flow profile was reached by a steady-state simulation, and it coupled to the transient simulation where the heat recovery can be calculated. The results of the coupled model are illustrated in **Fig. 1-2**.

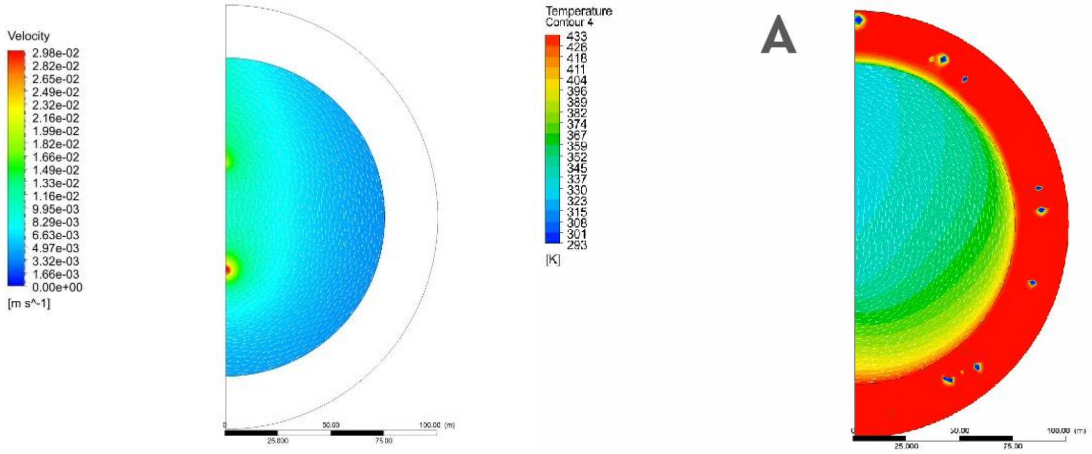

Figure 1-2: Results of the coupled model (left: flow profile reached by the steady-state simulation, right: temperature profile reached by the transient simulation)

After the numerical model was established the validation of the model was essential. For this purpose, several API standard measurements were performed and a new 3-dimensional model was built based on the apparatus. The measurements and the FEM results were compared.

The FEM simulation can require significant calculating capacities depending on the model and number of discrete elements. Since the dimensions of the fracture is several magnitudes smaller than the geothermal reservoir the developed model become robust and required several weeks to complete. To overcome this limitation an evolutionary algorithm-based optimization method was developed where the regression model resulted in a significant time reduction.

Numerous arrangements of the proppant-pack permeabilities can be reached in the fracture and all of them provide different thermal drawdown and production efficiency. To find the best arrangement of the zones (in the point of heat recovery) a response surface optimization method was developed with the integration of a multivariate polynomial regression.

The research techniques included all the conventional scientific techniques:

- literature review to identify perspectives,
- investigating analytical and numerical models to define research goals,
- semi-analytical model development to describe the phenomenon,
- numerical model development to represents transport phenomenon,
- performing laboratory experiments to validate the created numerical model
- utilize numerous optimization methods to make the most out of the models.

2 SCIENTIFIC ACHIEVEMENTS

Novel scientific results have been found during the research conducted and presented in the Ph. D. Thesis. This chapter summarizes the scientific findings and forms the theses.

2.1 Thesis #1

Proppant can be viewed as an unconsolidated, heterogeneous (in size distribution) although well-sorted granular packing, and the different relationships that can predict the permeability of such systems can be utilized. The pressure drop across a proppant-pack is usually described by the permeability term, while several correlations for granular-packings use the term of modified particle friction factor. In essence these terms try to describe the same phenomenon but the relationship between them was not identified in the literature as other approaches were used. To be able to analyze the different models a **novel relationship was established by the Author between the modified particle friction factor and the proppant-pack permeability** in the form of Eq. 2-1**.**

With the developed relationship (Eq. 2-1) the applicability of the different modified particle friction factor models can be extended. The two main driving mechanism in flow through porous media is the viscous shear from the fracture wall, and the viscous drag from the surface of the proppants. The modified particle friction factor models usually account for the viscous drag from the surface of the proppants and neglect the effect of the apparatus to the system. Complementing an analytical model that considers both mechanisms with the developed relationship (Eq. 2-1) **a new semi-analytical model was developed by the Author in the form of Eq. 2-2, with which the behavior of different models can be applied on proppant-packs**. The behavior of the different models with the new semi-analytical model clearly accounts for both mechanism as it can be seen in Fig. 2-1.

$$k_p = \frac{d_p \nu}{V f_p} \tag{2-1}$$

$$k_f = \left(\frac{d_p \nu}{V f_p}\right)\left\{1 + \frac{2}{w_f}\sqrt{\frac{\left(\frac{d_p \nu}{V f_p}\right)}{\phi_p}}\left[csch\left(\sqrt{\frac{\phi_p}{\left(\frac{d_p \nu}{V f_p}\right)}}w_f\right) - coth\left(\sqrt{\frac{\phi_p}{\left(\frac{d_p \nu}{V f_p}\right)}}w_f\right)\right]\right\} \tag{2-2}$$

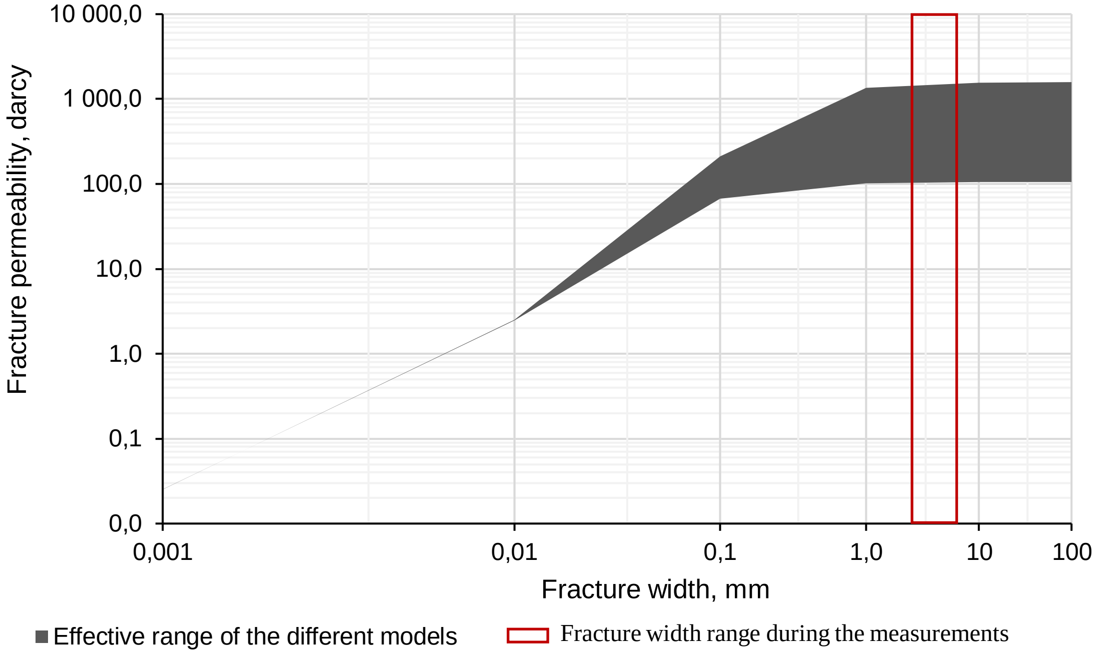

Figure 2-1: Effective range of the different modified particle friction factor models using Eq. 2-2

2.2 Thesis #2

Using Eq. 2-2 and the modified particle friction factor correlations presented in Tab. 2-1, the measured fracture permeability values can be compared with the correlation results. As f_p correlations contain the proppant-pack porosity (Φ_p) as a parameter, its values had to be determined. To calculate the porosity of the proppant-pack under different applied closing pressures, two parameters needed to be measured, which are the specific gravity of the proppant particle and the bulk density of the proppant-packs at each applied closing pressure.

A new method had to be developed to determine the bulk density of the proppant-pack under different applied closing pressures as no method can be found in the literature for this purpose. **A new measurement method was developed based on the** API RP 19C (2008) **standard method to determine the bulk density of proppant-packs under different closing pressures** and the schematic representation of the new method can be seen in Fig. 2-2. The results show that with an average 4.41% gross variance the bulk densities can be measured under different closing pressures. With the obtained bulk density values the porosities can be easily determined as it is illustrated in Fig. 2-3.

Table 2-1: Modified particle friction factor correlations considered in this study

Author(s)	Modified friction factor	Equation	Applicable range
Carman (Carman, 1937)	$f_p = \left(180 + 2.871\left(\frac{Re}{1-\Phi_p}\right)^{0.9}\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3 Re}$	(2-3)	$0.01 < Re_1 < 10{,}000$
Rose (Rose, 1945a)	$f_p = \left(\frac{1{,}000}{Re} + \frac{60}{\sqrt{Re}} + 12\right)h(\Phi_p)$	(2-4)	$0.01 < Re < 10{,}000$ $0.32 < \Phi_p < 0.9$
Morcom (Morcom, 1946)	$f_p = \left(\frac{784.8}{Re} + 13.73\right)\left(\frac{0.405}{\Phi_p}\right)^3$	(2-5)	$Re < 750$
Rose and Rizk (Rose and Rizk, 1949)	$f_p = \left(\frac{1{,}000}{Re} + \frac{125}{\sqrt{Re}} + 14\right)h(\Phi_p)$	(2-6)	$0.01 < Re < 10{,}000$
Leva (Leva, 1949)	$f_p = 2f_m\frac{(1-\Phi_p)^{(3-n)}}{{\Phi_p}^3}$	(2-7)	$Re < 10{,}000$
Fahien and Schiver (Fahien and Schiver, 1961)	$f_p = \left(q\frac{f_{1L}}{Re_m} + (1-q)\left(f_2 + \frac{f_{1T}}{Re_m}\right)\right)\frac{(1-\Phi_p)}{{\Phi_p}^3}$	(2-8)	No data
Tallmadge (Tallmadge, 1970)	$f_p = \left(150 + 4.2\left(\frac{Re}{1-\Phi_p}\right)^{5/6}\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3 Re}$	(2-9)	$0.1 < Re_m < 100{,}000$
Macdonald et al. (Macdonald et al., 1979)	$f_p = \left(180 + 1.8\left(\frac{Re}{1-\Phi_p}\right)\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3 Re}$	(2-10)	$Re_m < 10{,}000$
Foscolo et al. (Foscolo et al., 1983)	$f_p = (17.3 + 0.336Re)\frac{(1-\Phi_p)}{{\Phi_p}^{4.8} Re}$	(2-11)	No data
Meyer and Smith (Meyer and Smith, 1985)	$f_p = \left(90 + 0.462\left(\frac{Re}{1-\Phi_p}\right)\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^{4.1} Re}$	(2-12)	$Re_1 < 1{,}000$
Watanabe; Kurten et al.; Steinour (Watanabe, 1989; Kurten et al., 1966; Steinour, 1944)	$f_p = 6.25\left(\frac{21}{Re} + \frac{6}{\sqrt{Re}} + 0.28\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3}$	(2-13)	$0.1 < Re < 4{,}000$
Avontuur and Geldart (Avontuur and Geldart, 1996)	$f_p = \left(141 + 1.52\left(\frac{Re}{1-\Phi_p}\right)\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3 Re}$	(2-14)	$Re_m < 10{,}000$
Erdim et al. (Erdim et al., 2015)	$f_p = \left(160 + 2.81{Re_m}^{0.904}\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^3 Re}$	(2-15)	$2 < Re_m < 3{,}600$

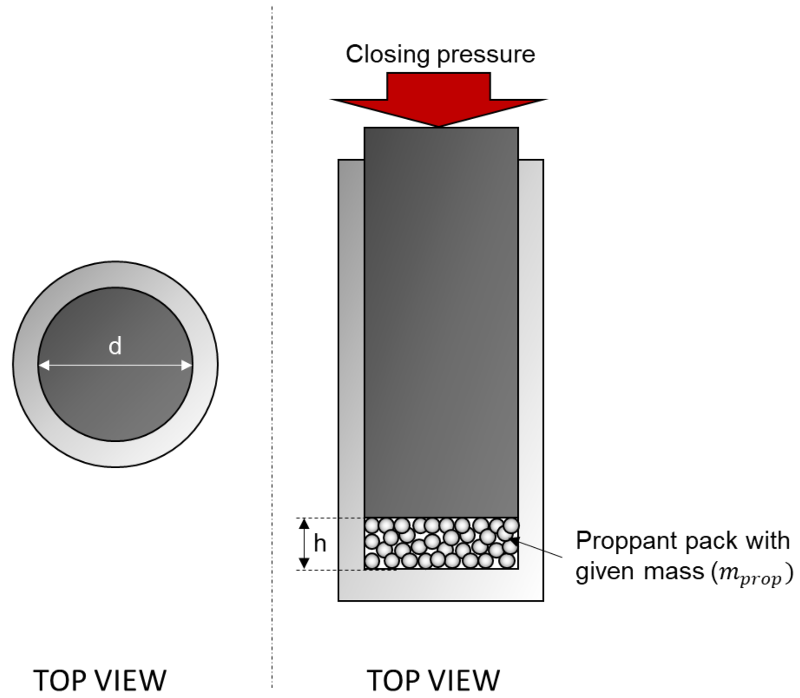

Figure 2-2: Schematic illustration of the crush test apparatus used during the bulk density measurements (not to scale)

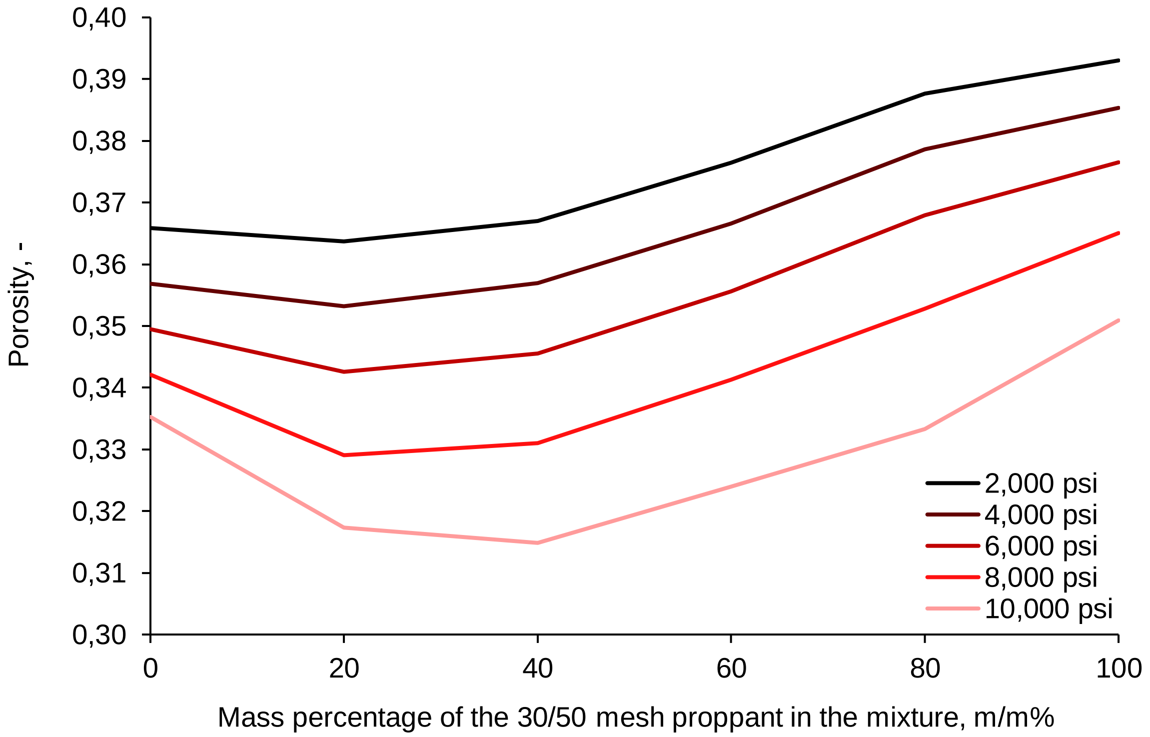

Figure 2-3: Porosity values at different applied closing pressure in case of different mixtures

2.3 Thesis #3

The different modified particle friction factor correlations do not describe the behavior of proppant-pack under reservoir conditions efficiently, as it is shown in **Fig. 2-4**. This can be interpreted by the following considerations:

- In the case of the models represented in **Tab. 2-1** the particles were unconsolidated, where the particles usually can move nearly freely, while during the proppant measurements, the proppant particles were nearly in fixed position.
- In the case of the models represented in **Tab. 2-1** there was no applied pressure on the particles, while during the measurements, the particles were under significant pressure, which can cause deformation, crushing, and embedment on the closure body‘s surface. These effects can significantly deteriorate the final permeability results (Liang et al., 2016).

A correction was applied on the available models to overcome these effects and provide a new, more effective correlation by implementing a pressure dependent term. The pressure dependent term was considered to be an exponential relationship and **Eq. 2-3** was developed to account for this effect. The correlation coefficients were optimized to best fit to the measurement values and **a new modified particle friction factor correlation was developed which account for the closing pressure that is asserted on the particles in the form of Eq. 2-4.** With this new correlation the average gross variance of 12.4% can be reached compared to the 34.1%-100% of other correlations. The behavior of this new correlation compared to the measurement values can be seen in **Fig. 2-5**.

$$f_p = e^{\alpha\frac{P_c}{1{,}000}}\left[\left(a + b\left(\frac{Re}{1-\Phi_p}\right)\right)\frac{(1-\Phi_p)^2}{{\Phi_p}^c Re}\right] \tag{2-3}$$

$$f_p = e^{0.203\frac{P_c}{1{,}000}}\left[\left(542 + 0.462\left(\frac{Re}{1-\Phi_p}\right)\right)\frac{(1-\Phi_p)^2}{\Phi_p Re}\right] \tag{2-4}$$

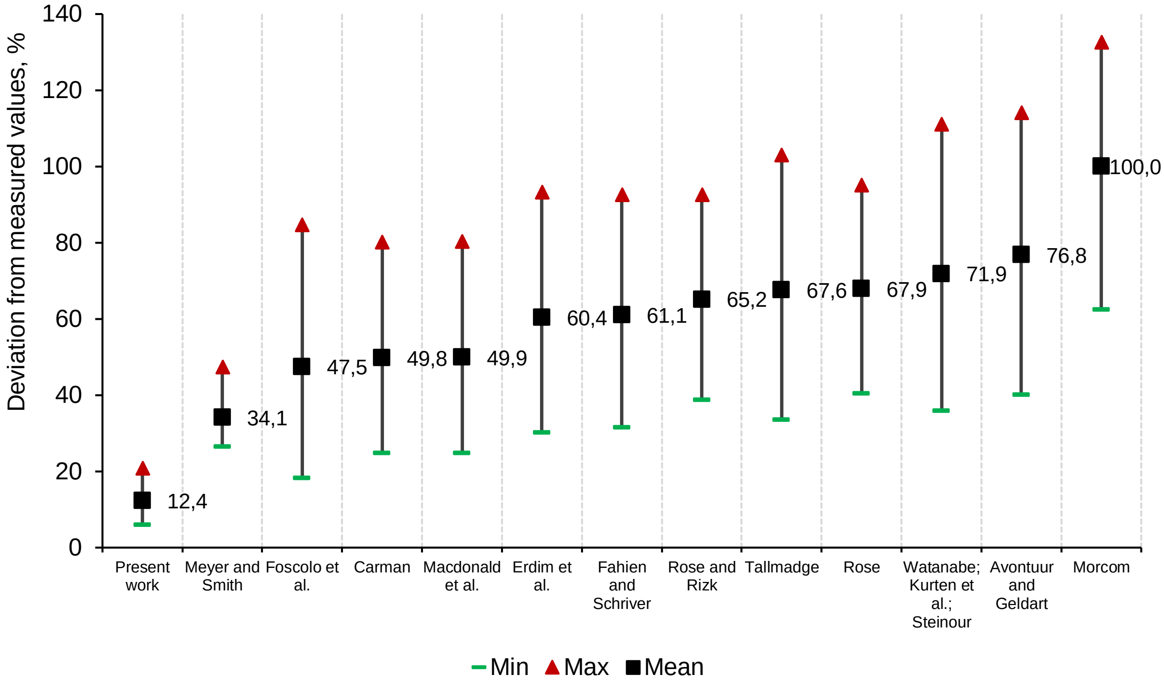

Figure 2-4: Comparison of the calculated and measured fracture permeability values in case of the different models

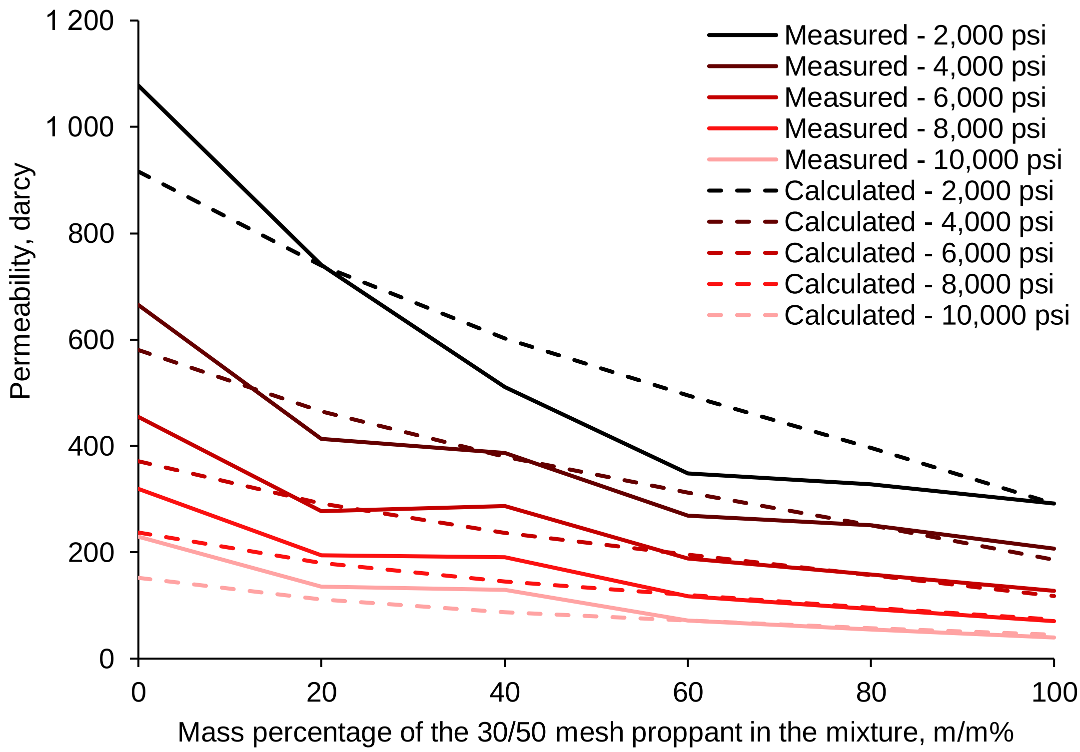

Figure 2-5: Comparison of the present work to the measured fracture permeability values

2.4 Thesis #4

An optimization method was developed with which the time required to simulate the transient heat recovery of the system can be drastically reduced. The optimization method is based on an evolutionary algorithm and the results show that after around 90 days (simulation time) the remaining exponential curve of the temperature decrease can be estimated under 0.5% deviation as it can be seen in Fig. 2-6. It was tested on 5 different simulation and one detailed example is shown in Fig. 2-7.

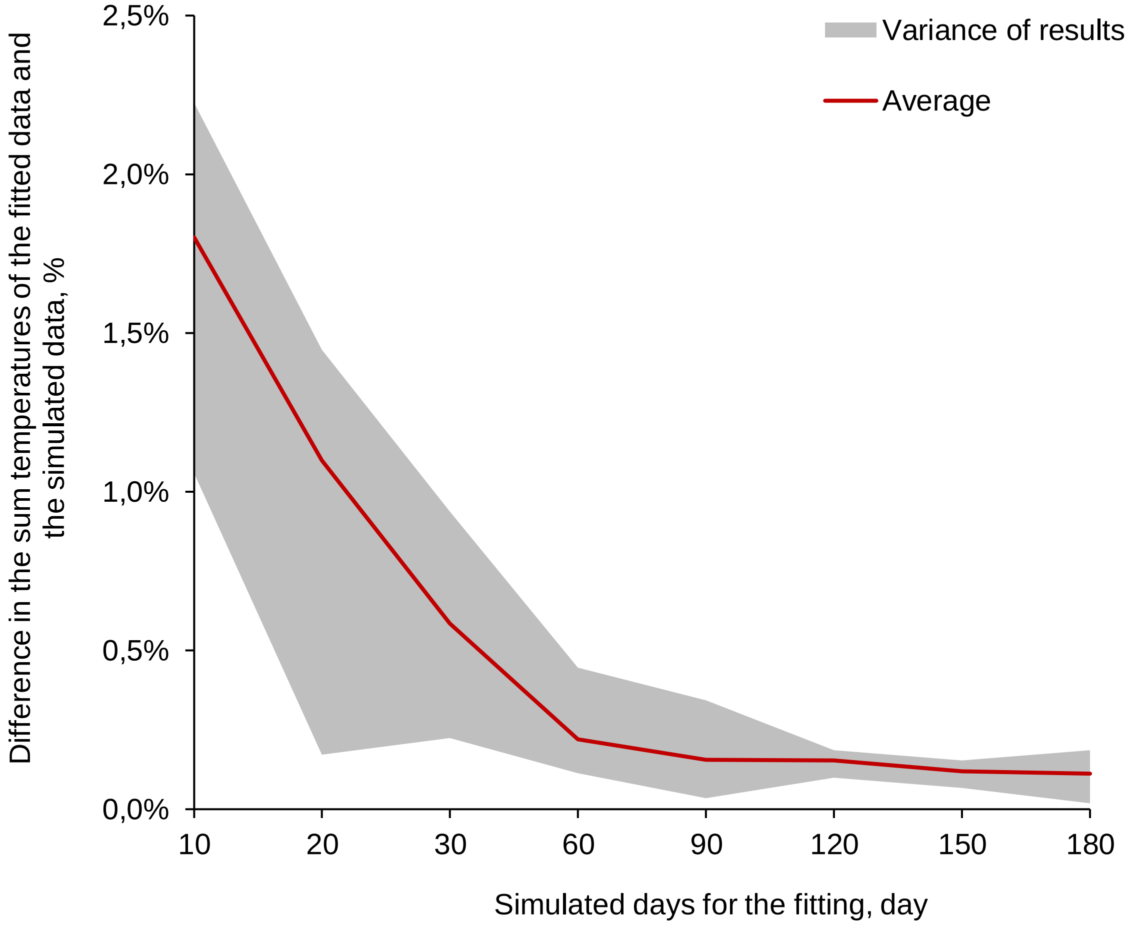

Figure 2-6: Result of the curve fitting to reduce simulation time

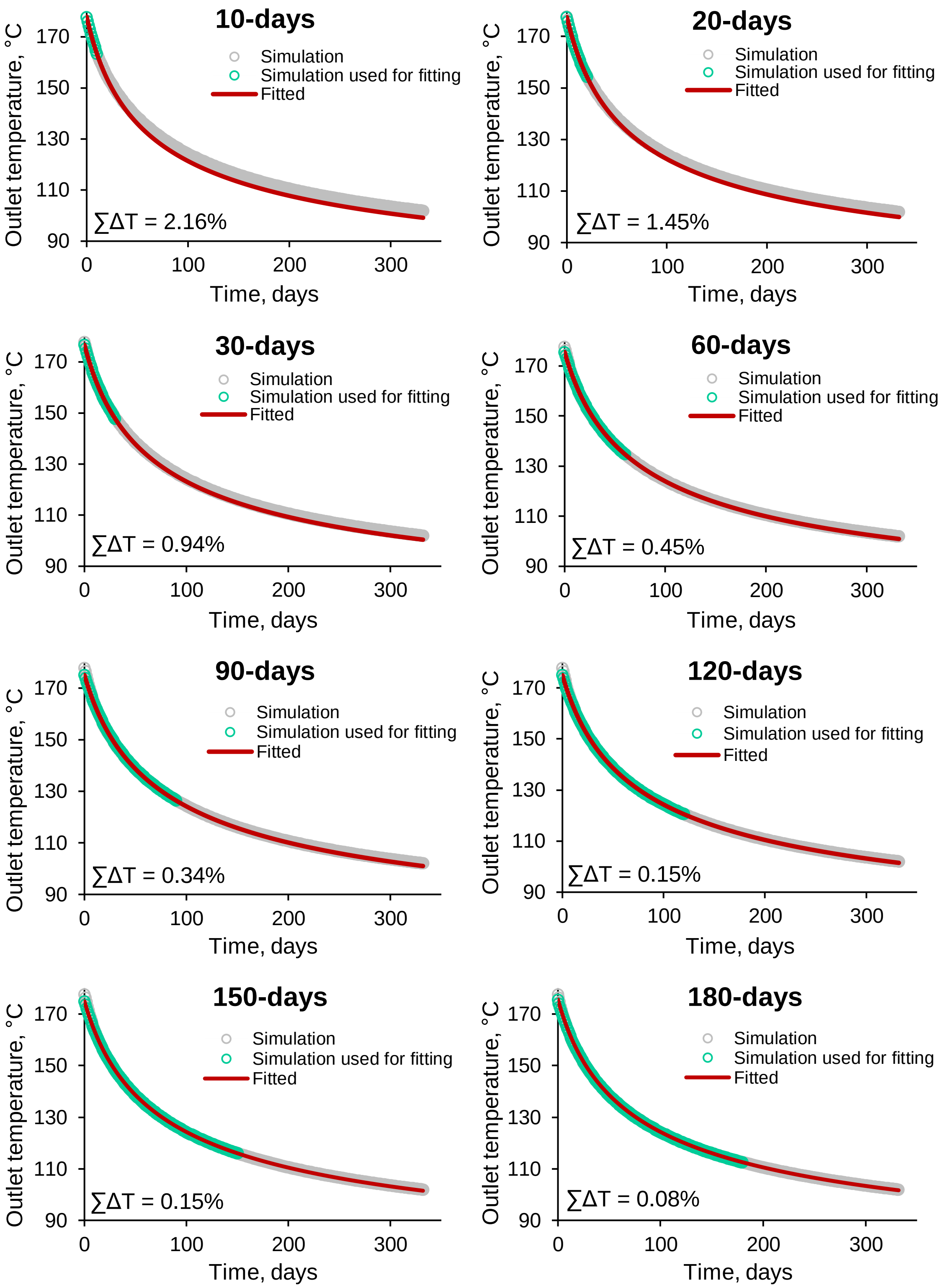

Figure 2-7: Fitting results based on different simulation time in case the fracture permeability in the zones is 128 *Darcy* with the difference in the sum temperatures ($\sum \Delta T$) of the simulated and the fitted data

2.5 Thesis #5

Most of the EGS methods involve one or more injection and production wells to be implemented. Later studies such as Kehrer et al. (2007) and Danko et al. (2018) offers EGS systems where the injection and production are integrated in one well, thus reducing the cost of the total investment. These methods although have some advantages also have some limitations that promotes the investigation of a novel technology.

It was proven with simulations by the Author that the flow inside a propped fracture can be controlled by creating concentric zones with different permeabilities. Significant differences can be reached in the heat recovery by this method as it is shown in **Fig. 2-8**.

A new EGS method was proposed by the Author with which a more efficient heat recovery can be reached. In this new method a propped fracture is used where different concentric zones are established. A complex Finite Element Model was established by the Author to simulate the new EGS method. The developed 3-dimensional model can be analyzed by a state-of-the-art method where the steady-state simulation is coupled to the transient simulation. This coupling method facilitates to reduce the simulation time, while the model accuracy is not harmed. This could be reached as the fluid flow inside the fracture will reach a steady state after enough time (this can be seen in **Fig. 2-9**), while the heat transfer from the reservoir to the fluid is constantly changing as the circulated fluid draws the reservoir temperature down (illustrated in **Fig. 2-10**). The validity of the model was proved by several standard measurements, where the results of the simulation showed an excellent match with the measured data, and it can be seen in **Fig. 2-11**.

An optimization procedure was also established by the Author where the new EGS technology can be optimized in the point of heat recovery. The optimization method is based on a Response Surface Method (RSM) using multivariate polynomial regression (MPR). The results proved that there is an optimal arrangement of the zones' permeability values where the maximum heat recovery can be reached. The RSM optimization method is illustrated in **Fig. 2-12**, where the dependent variables are the permeability values of the zones and the independent variable that should be maximized is the sum of outlet temperature values of the different simulations.

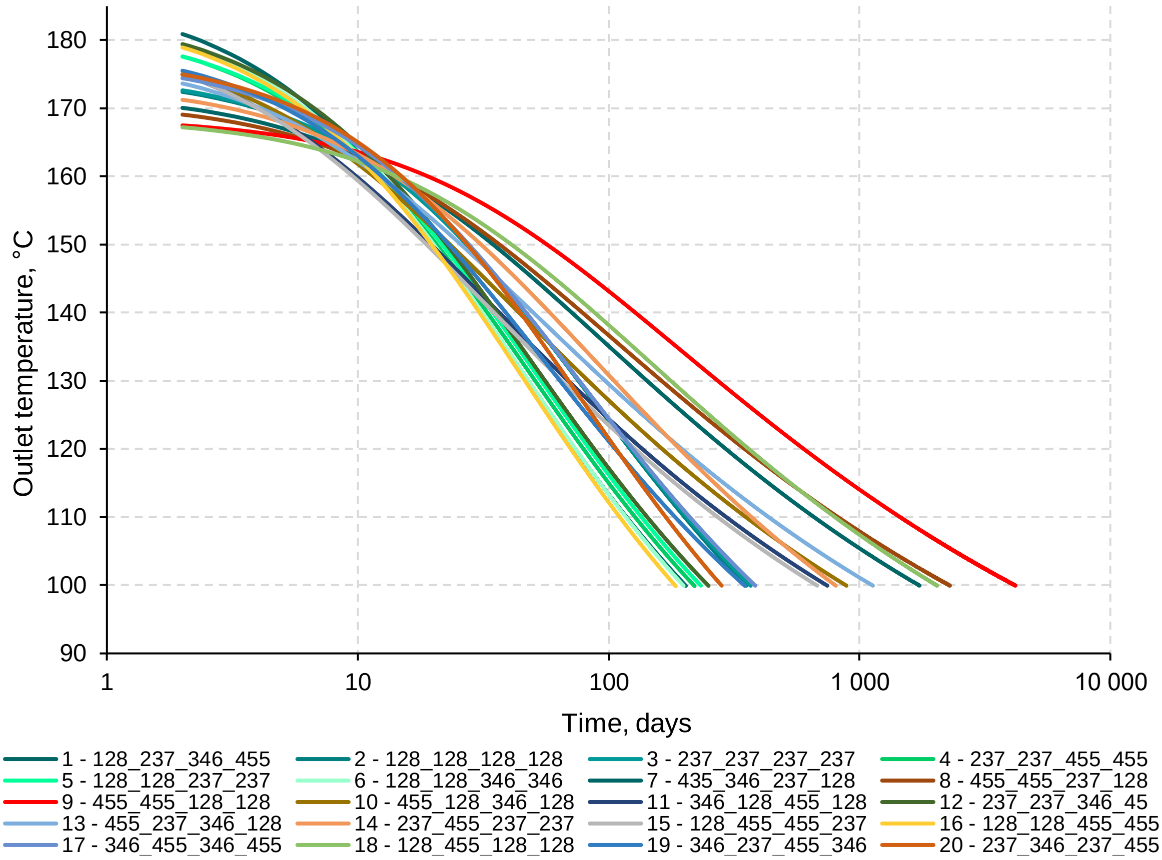

Figure 2-8: Results of the simulations

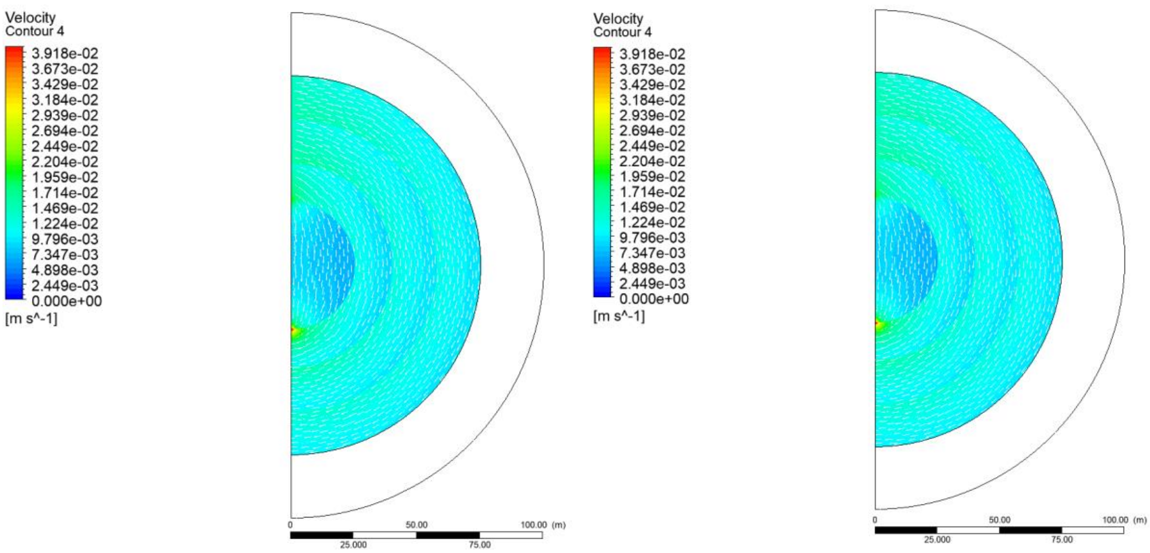

Figure 2-9: Flow profiles (left with 60°C inlet temperature and right 180°C inlet temperature)

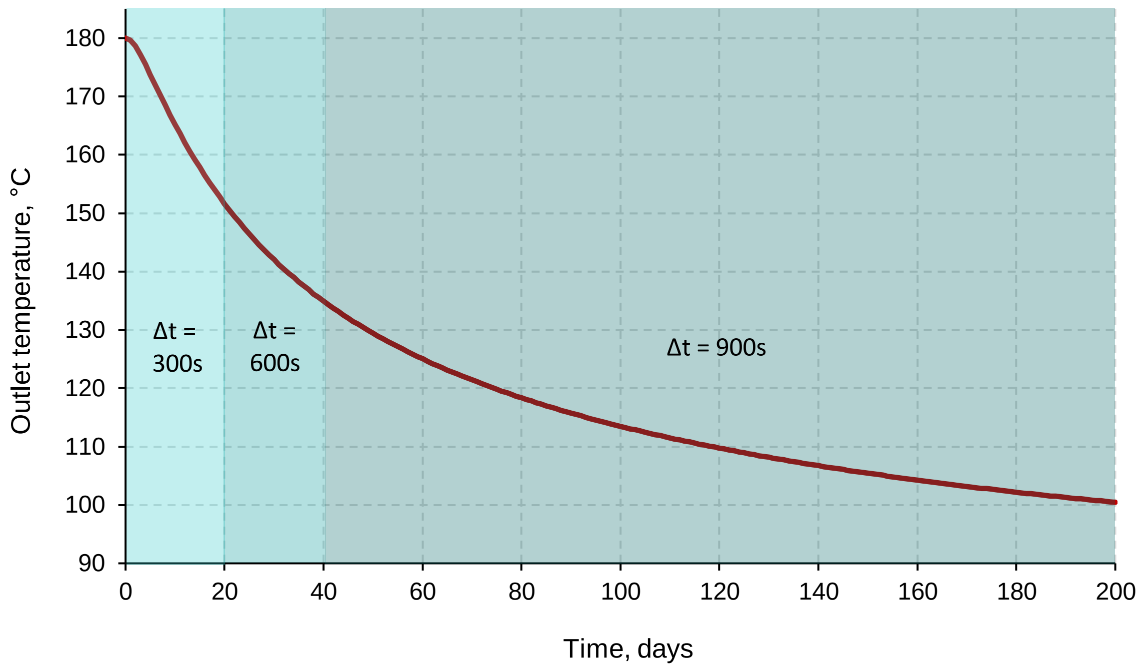

Figure 2-10: Sensitivity analysis for transient timestep

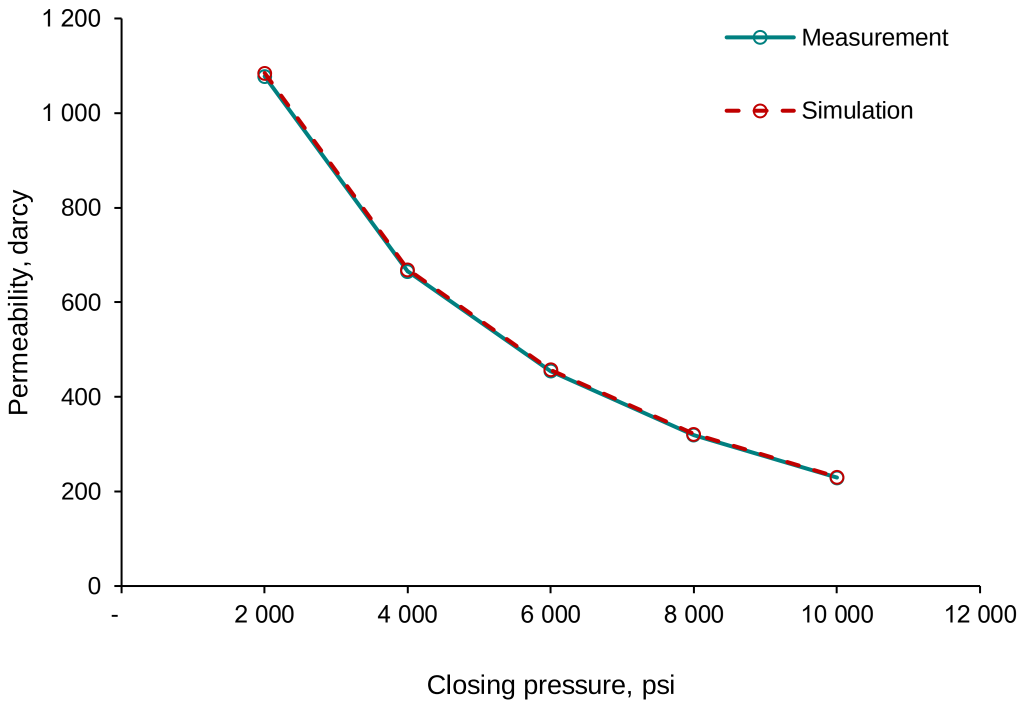

Figure 2-11: Results of the validation

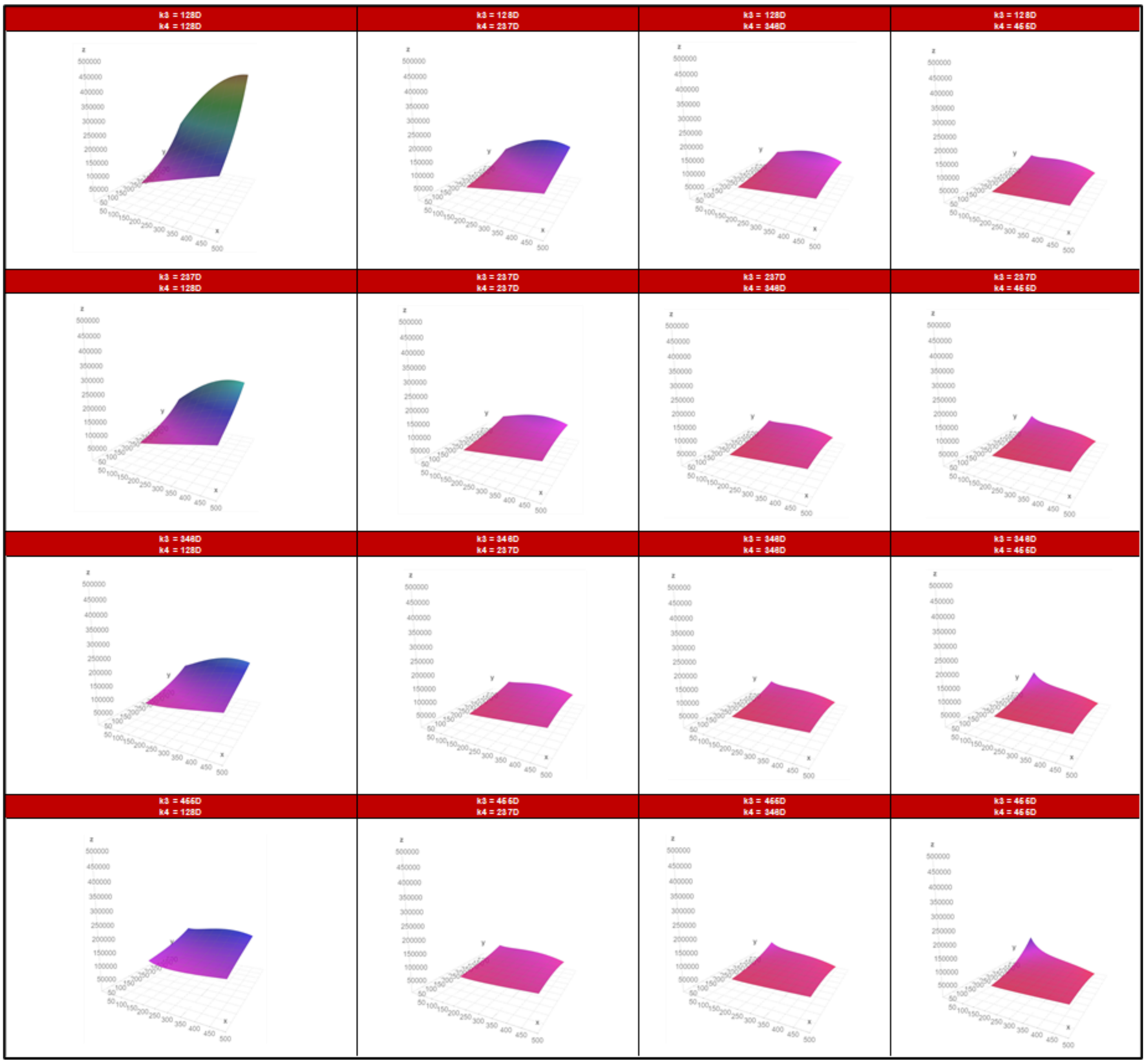

Figure 2-12: Response surface of the analyzed dataset

3 SUMMARY

Currently, Enhanced Geothermal Systems do not play a significant role in the renewable energy mix. In the past few years, several advancements were reached but the forecasts clearly indicate that although geothermal energy production has a major potential its widespread utilization is not probable without a compelling technological enhancement.

The research conducted and presented in the Thesis developed a novel EGS technology and established its theoretical background. First, a new idea was presented where a single-well, hydraulically fractured EGS system is defined where the fracture is supported by proppant. The fracture was divided into 4 circular zones in which the proppant permeability, thus the fracture permeability could be arbitrarily determined. With this method the deficiencies of the currently investigated similar approaches can be avoided, namely the whole rock surface is available for heat transfer and the different permeability zone provide an opportunity to control the flow in the fracture. A 3-dimensional Finite Element Model was built for simulating the new model. As a state-of-the-art approach, the steady-state and transient simulation was coupled in the model that facilitated to decrease in the required computational capacity. The model was validated by several API standard measurement that was carried out by the Author.

The research showed that the fluid flow in the propped fracture can be controlled by creating circular zones with different permeability values. This result proved the concept and open the possibility to optimize the permeability arrangement in the fracture to reach the best heat recovery. For the optimization purpose, a modern approach was designed based on a Response Surface Method applying multivariate polynomial regression. After the optimization, it was shown that the proposed technology if designed well can provide around 20 times better heat recovery from a geothermal resource than other models.

The research demonstrated that by mixing two proppant-packs the optimized fracture permeability values can be reached. For this, the Author performed several measurements and developed a new relationship between the modified particle friction factor and the proppant-pack permeability. With this new relationship, a semi-analytical model could be defined where the two driving mechanisms during flow through porous media could be integrated. A new bulk density measurement was also elaborated with which the proppant-pack porosities could be identified under different applied closing pressure.

The outcome of the research is a new EGS technology that can facilitate further studies and advancement in the renewable energy industry.

4 THE AUTHOR'S MAIN SCIENTIFIC PUBLICATIONS

4.1 Publications

Jobbik, A., Lengyel, T. & Pusztai, P. (2015). *Összetett matematikai modell hidraulikus rétegrepesztés optimalizálására*. Műszaki Földtudományi Közlemények, University of Miskolc, Miskolc, 85(1):97-105, ISSN 2063-5508

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Pusztai, P. & Jobbik, A. (2021). *Investigation of gas flow models in case of micro and nano pore size reservoirs*. Geosciences and Engineering, Miskolc University Press., Miskolc, 9(14):96–115, ISSN 2063-6997

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