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**Thesis of the doctoral dissertation (PhD)**

**INVERSION-BASED METHOD DEVELOPMENT TO IMPROVE THE  
INTERPRETATION OF POTENTIAL FIELD GEOPHYSICAL DATA**

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## **I. SCIENTIFIC BACKGROUND AND AIMS**

The Fourier transformation is widely regarded as one of the most commonly utilized tools in geophysical data processing. In signal processing, several Fourier transformation algorithms methods are provided to enhance the quality of the geophysical datasets being interpreted, and hence, a comprehensive picture of the subsurface geology can be effectively drawn. It is well known that geophysical datasets collected in the field might occasionally encounter a slew of problems. This is owing to the presence of obstacles, either naturally or purposefully manufactured, in the surface area being investigated. In particular, advances in geophysical equipment as well as various surface topographical characteristics, like caverns, hills, and mountains, have a significant impact on data qualification. The constraints that necessitate the survey to be carried out on non-regular or random grids of measuring data points. In that case, it is quite likely that a fraction of noise outliers unrelated to subsurface geological anomalies will be included in the geophysical information gathered. Additionally, the random surveying of the areas under investigation causes gaps in missing datasets, and therefore, a significant amount of information is susceptible to loss. Due to the inefficiency and limitations of the traditional Fourier transformation (DFT) approach in processing the outlier noisy data as well as the incomplete sampling designs, this thesis introduces an algorithm of the inversion-based 1D and 2D Fourier transformation (IRLS-FT) to cope with these challenges. Dobróka et al (2015) used a one-dimensional (1D) inversion technique, which was further extended and adapted to two-dimensional (2D) inversion (Dobróka et al, 2017). Their research findings revealed sufficient improvements in both the space and frequency domains.

In the framework of this inversion-based Fourier transformation method, the continuous Fourier spectrum is discretized using the series expansion method to

solve our over-determined inverse problem in the form of the expansion coefficients (Szabó, 2011 and 2015; Turai and Dobróka, 2011). Moreover, the Hermite functions are constructed as basis functions, taking advantage of the fact that they are eigenfunctions of the Fourier transformation, permitting quick and precise calculation of the elements of the Jacobian matrix. To make the inversion algorithm more robust and resistant, the Cauchy weights (Amundsen, 1991) are often integrated into an Iteratively Reweighted Least Square (IRLS) algorithm (Scales et al, 1988), but in this case, the computation of the scale parameters is problematic because they should be known in advance. As a result, the most frequent value (MFV) method (Steiner, 1988 and 1997) is used to handle this problem by iteratively calculating the Cauchy Steiner weights through an internal iteration loop instead of Cauchy weights in a manner that minimizes data loss. In this study, MATLAB-based codes for the proposed methods of conventional discrete Fourier transform (2D DFT) and inversion-based Fourier transformation (2D IRLS-FT) are implemented on synthetic 1D wavelets, synthetic 2D magnetic and gravity datasets, and real gravity field data measurements. The aim of my study is to reduce the outlier sensitivity using the IRLS inversion-based FT compared to the traditional discrete Fourier transform (DFT). Furthermore, the inversion algorithm is applied to solve the non-regular sampling problem-based complete datasets in 1D and 2D. The final objective is the processing of the incomplete or missing datasets sampled both equidistantly and non-equidistantly utilizing our newly developed inversion approach.

## **II. ACCOMPLISHED INVESTIGATION**

In accordance with the main objectives of my thesis, I tested the inversion-based Fourier transformation (IRLS-FT) method on synthetic datasets sampled in one dimension. A time-domain 1D equidistant synthetic Morlet wavelet is constructed

over 401 measurement points, which is subsequently contaminated with both Gaussian and random noise of Cauchy distribution, emulating the real filed data measurements. For noise sensitivity, I found that the inversion-based Fourier transformation (IRLS) approach is resilient and has a substantial ability to reduce outlier spread when compared to those found by the traditional DFT method, as numerically evidenced by the data and model distance values. For solving the non-regular sampling problem by our inversion method, I randomized the datasets over two configurations; random measurement positioning and random walk sampling. The results provide extreme effectiveness, robustness, and applicability of the inversion method for processing 1D complete datasets gathered in non-regular configurations. Furthermore, I have developed the inversion algorithm to be applied to the 1D incomplete or missing datasets sampled both regularly and non-regularly, and it is effectively extended to the incomplete block sampling designs. Missing 15%, 30%, and 50% of the measurements are used in both regular and non-regular sampling cases. In both cases, I demonstrated that the new inversion approach is highly appropriate for processing both regularly and non-regularly collected incomplete datasets, even when half of the observations (50%) are missing. In addition, the inversion method is highly effective in solving the block incomplete sampling problem.

On the other hand, for noise rejection analyzing purposes, equidistantly sampled 2D noise-free and noisy synthetic magnetic datasets are subjected to the above-mentioned noise filtering-based 2D Fourier transformation techniques in the framework of reduction to the pole. In my calculations, I used a total of 1681 measurements and Hermite functions of order ( $M_x=M_y=25$ ) are chosen. The inversion method is also tested on the non-regular complete magnetic datasets sampled with the same two configurations as 1D data. Accordingly, I have found that the inversion method has a higher noise rejection capacity, as well as its

effectiveness in handling the non-regular sampling problem. The newly developed 2D inversion algorithm is employed for treating the incomplete sampling problem of the 2D magnetic datasets. Hence, regularly, non-equidistantly as well as randomly sampled magnetic datasets are all subjected to the inversion algorithm using different missing percentages (15%, 25%, 35%, 45%, 50%, and 60%). According to the results, I have demonstrated that the inversion gives adequate improvements up to 50% of missing data as a maximum, far beyond this point, the inversion is ineffective because our inverse problem is sounded to be marginally over-determined where the observing data points used are nearly close to the number of the model parameters ( $M_x M_y=625$ ).

Furthermore, both the traditional DFT and IRLS inversion-based FT methods are tested and evaluated on 2D noisy and noise-free synthetic gravitational datasets generated using a model of a right rectangular prism. The model is initially applied to a regular grid before being randomized to produce irregularly sampled gravity intervals. I have introduced new geophysical applicability in the field of low-pass filtering of such 2D gravitational datasets. In both configurations, the inversion procedures are executed on 1681 measurements using Hermite functions of order  $M_x=M_y=25$  as an inversion parameter for a compromise between accuracy and stability. The results demonstrate the improved noise reduction capability of the inversion-based Fourier transformation algorithm. The inversion efficiency and stability in processing gravitational data sampled non-equidistantly are also established. Besides, the 2D incomplete noise-free and noisy gravity data are then comprehensively subjected to the newly proposed inversion-based Fourier transformation algorithms on an equidistantly and non-equidistantly sampled basis. As in the magnetic methodology, I used missing data percentages of (15%, 25%, 35%, 45%, 50%, and 60%). In my investigation, I found that stability and effectiveness of the inversion used are evidenced for all missing data below 50

percent (the percentage at which the inverse problem is completely over-determined) which are distorted after that point due to the same problem of marginal over-determination ratio.

Finally, I introduced a field example as a case study in the western central part of Sinai Peninsula, Egypt. In that regard, I evaluated the noise sensitivity reduction of the low-pass filtering-based Fourier transformation on real equidistantly and non-equidistantly sampled gravitational field datasets (441 measurements). Hence, I used Hermite functions of order  $M_x=M_y=13$  to estimate the regional gravity anomaly maps and surfaces. The results showed the same properties reflecting the inversion's efficacy and robustness for handling both the equidistantly and non-equidistantly measured field datasets. Such real gravity field datasets measured over both equidistant and non-equidistant grids are also involved to assess the incomplete or missing sampling problem using our newly developed inversion-based low-pass filtering. I have chosen missing data percentages of 25%, 35%, 50%, and 60% to accomplish the inversion stability. Accordingly, I demonstrated that as the missing data percentages increase after 50% in both equidistant and non-equidistant sampling cases, the obtained results are distorted where the number of the field data measuring points is nearly close to the number of the model parameters ( $M_x M_y=169$ ), and hence the inverse problem is slightly overdetermined.

### **III. NEW SCIENTIFIC RESULTS**

#### **Thesis 1**

I gave a comprehensive analysis of the inversion-based Fourier transformation algorithm applied to 1D synthetic wavelet to reduce the outlier sensitivity as well as to deal with the issues of irregular sampling and incomplete missing data. To

achieve such aims, I used a time-domain signal created over 401 measuring points which are then contaminated with both Gaussian and Cauchy distributed noises.

Dealing with the regularly and non-regularly sampled incomplete datasets, I found that the inversion is highly effective, robust, and applicable even when half of the measurements (50%) are missing. Far beyond this percentage, the inversion algorithm becomes unstable and does not perform effectively where the inverse problem is marginally over-determined. In my analysis of the block-incomplete sampling problem, I found that for certain block intervals, the accuracy of the 1D datasets in both frequency and time domains is well satisfactory which is highly distorted as I increase the missing block intervals because of the marginal over-determination rate.

## **Thesis 2**

In the field of pole reduction of magnetic data, I gave an extended analysis of the 2D IRLS-FT method with regard to the outlier sensitivity, the non-regular sampling, and the missing data problems in a numerical example containing  $41 \times 41$  measurement points and  $25 \times 25$  unknown expansion coefficients.

Studying the missing data problem on noise-free and Cauchy noise-contaminated datasets (containing outliers), I found that in the case of regularly sampled, non-regularly sampled, and random-walk datasets, increasing the ratio of the missing data up to 50%, the accuracy of the pole reduced magnetic map is acceptable, while at higher ratios (when the over-determination rate is below 1.3) the results are highly distorted.

## **Thesis 3**

I developed a new filtering procedure (low-pass filtering) based on the 2D IRLS-FT method and applied it to gravity data. I constructed a model of a right

rectangular prism to generate 2D synthetic gravity datasets which are initially created over an equidistant grid before being randomized to provide a random-walk sampling design. In my calculations, the inversion procedures are executed using Hermite functions of order  $M_x=M_y=25$  as an inversion parameter for a compromise between estimation accuracy and stability. According to the results on both space and frequency domains, I proved that the inversion-based filtering procedure has a highly reduced outlier sensitivity in the cases of regular and random-walk sampled datasets.

#### **Thesis 4**

I gave an extended analysis of the 2D IRLS-FT-based filtering method in terms of the missing gravity data problems in a numerical example containing  $41 \times 41$  measurement points and  $25 \times 25$  unknown expansion coefficients.

In my analysis of the missing data filtering problem on noise-free and Cauchy noise-contaminated gravity datasets (containing outliers), I found that the inversion algorithm is stable and highly applicable for handling both the equidistantly and random-walk sampled incomplete gravity datasets even when 50 percent of the data measurements are cancelled or missing. In contrast, distorted anomaly shapes and amplitudes are provided at higher rates of the missing data percentages (when the over-determination rate is below 1.3).

#### **Thesis 5**

I assessed the efficiency and accuracy of the 2D IRLS-FT inversion method in outlier sensitivity and missing data problem using a field example of real gravity measurements sampled both equidistantly and non-equidistantly in the western central part of Sinai Peninsula, Egypt. A total of 441 measuring points were

carried out over a grid of 21x21, whereas I set the unknown expansion coefficients as 13x13 to match the inversion procedure's accuracy and stability.

In the framework of the low-pass filtering, I found that the inversion method has a higher capacity for noise rejection and gave me similar regional gravity anomaly maps reflecting the inversion's efficiency and robustness for processing both the equidistantly and non-equidistantly measured field datasets. In the case of a missing data problem, I found that measuring only half of the study area's total data capacity (50%) is sufficient to present the inversion stability and accuracy, whether I treat equidistantly or non-equidistantly sampled field data measurements. The results are distorted at the missing percentages over 50%, where the over-determination rate decreases below 1.3.

#### **IV. PRACTICAL APPLICATION OF RESULTS**

In the framework of my Ph.D. dissertation, I performed geophysical inversion method development for data processing. My dissertation provides the necessary mathematical formulations and algorithms applied for geophysical data processing and presents the physical properties derived from the analysis of magnetic and gravity potential data. The developed algorithms can be considered advanced and efficient methods in the inversion-based Fourier transformation. These methods have sufficient noise suppression capability and outlier resistance, therefore there are possibilities of applications in the geophysical data processing and interpretation as well as other areas where Fourier transform is applied for the processing of noisy data. It is well known that most of the geophysical datasets are carried out over non-equidistant grids due to several restrictions either naturally or artificially constructed. It raises a more complicated or ill-posed problem of the missing data, and hence a significant amount of information is susceptible to loss. Therefore, the newly developed inversion-based Fourier transformation approach

is highly applicable and robust, opening up a new and economically powerful way of handling the incomplete sampling problems encountered during wide and diverse field data acquisition.

## V. REFERENCES

Amundsen, L. (1991). Comparison of the least-squares criterion and the Cauchy criterion in frequency-wavenumber inversion. *Geophysics* 56, pp. 2027-2035.

Dobróka, M., Szegedi, H., Vass P. (2017). Inversion-based Fourier transform as a new tool for noise rejection, INTECH 2017. DOI: <https://doi.org/10.5772/66338>

Dobróka, M., Szegedi, H., Somogyi Molnár, J. Szűcs, P. (2015). On the Reduced Noise Sensitivity of a New Fourier Transformation Algorithm. *Mathematical Geosciences.*, 47 (6), pp. 679-697. DOI: <https://doi.org/10.1007/s11004-014-9570x>

Scales, J. A., Gersztenkorn, A., Treitel, S. (1988). Fast Lp solution of large, sparse, linear systems: Application to seismic travel time tomography. *Journal of Computational Physics* 75, pp. 314- 333.

Steiner, F. (1988). Most frequent value procedures (a short monograph). *Geophysical Transactions*,34, pp.139-260.

Steiner, F. (1997). *Optimum Methods in Statistics*. Budapest: Academic Press.

Szabó, N. P. (2015). Hydraulic conductivity explored by factor analysis of borehole geophysical data. *Hydrogeol J*, 23(5), pp. 869–882. DOI: <https://doi.org/10.1007/s10040-015-1235-4>

Szabó, N. P. (2011). Shale volume estimation based on the factor analysis of well-logging data. *Acta Geophys*, 59 (5), pp. 935–953. DOI: <https://doi.org/10.2478/s11600-011-0034-0>

Turai, E., Dobróka, M. (2011). Data processing method developments using TAU transformation of time-domain IP data I. Theoretical basis. *Acta Geodaetica et Geophysica Hungarica*, 46 (3), pp. 283-290. DOI: <https://doi.org/10.1556/AGeod.46.2011.3.1>

## **VI. LIST OF RELATED PUBLICATIONS AND PRESENTATIONS**

### **JOURNAL ARTICLES**

1- Abdelaziz, M. I., Dobróka, M. (2020). Testing the noise rejection capability of the inversion based Fourier transformation algorithm applied to 2D synthetic geomagnetic datasets. *Geoscience and Engineering*, 8 (13), pp. 72-80. ISSN 2063-6997

2- Shebl, A., Abdellatif, M., Hissen, M., Abdelaziz, M. I., Csámer, Á. (2021). Lithological mapping enhancement by integrating Sentinel 2 and gamma-ray data utilizing support vector machine: A case study from Egypt. *International Journal of Applied Earth Observation and Geoinformation*, 105 (2021) 102619. DOI: <https://doi.org/10.1016/j.jag.2021.102619>

3- Shebl, A., Abdelaziz, M. I., Ghazala, H., Arafa, S. A. S., Abdellatif, M., Csámer, Á. (2022). Multi-criteria ground water potentiality mapping utilizing remote sensing and geophysical data: A case study within Sinai Peninsula, Egypt. *J. Remote Sensing Space Sci*, 25 (3), pp. 765–778. DOI: <https://doi.org/10.1016/j.ejrs.2022.07.002>

4- Abdelaziz, M. I. M., Vass, P., Dobróka, M. (2022). Az Inverziós Fourier transzformáció néhány meglepő tulajdonsága. *Magyar Geofizika*, in press.

### **CONFERENCE PAPERS (SHORT AND EXTENDED ABSTRACT)**

1- Abdelaziz, M. I. (2019). A contribution of the magnetic field data and geoelectrical resistivity sounding for groundwater exploration in a part of central

Sinai, Egypt. 50th Meeting of young Geoscientists, Abstract book, English language international conference abstract, pp. 32.

2- Abdelaziz, M. I. (2019). Application of magnetic and geoelectrical resistivity sounding methods in groundwater exploration in a part of central Sinai, Egypt. MultiScience – XXXIII. microCAD International Multidisciplinary Scientific Conference. English language international conference paper (ISBN 978-963-358-177-3). DIO:10.26649/music.2019.079

3- Abdelaziz, M. I., Dobróka, M. (2022). Inversion-based 1D and 2D Fourier transformation algorithm for solving the incomplete sampling problem. GeoMATES 2022, International Congress on Geomathematics in Earth-& Environmental Sciences, Abstract book, English language international conference abstract, pp. 71.

## **INTERNATIONAL CONFERENCE PRESENTATIONS**

1- A contribution of the magnetic field data and geoelectrical resistivity sounding for groundwater exploration in a part of central Sinai, Egypt. 50th Meeting of young Geoscientists, conference presentation, Ráckeve, Hungary. March 29-30, 2019.

2- Application of magnetic and geoelectrical resistivity sounding methods in groundwater exploration in a part of central Sinai, Egypt. MultiScience – XXXIII. microCAD International Multidisciplinary Scientific Conference, conference presentation, Miskolc, Hungary, May 23-24, 2019.

3- Inversion-based 1D and 2D Fourier transformation algorithm for solving the incomplete sampling problem. GeoMATES 2022, International Congress on Geomathematics in Earth-& Environmental Sciences, conference presentation, Pécs, Hungary, May 19-21, 2022.

**DOMESTIC CONFERENCE PRESENTATIONS**

1- Interpretation of magnetic and geoelectric resistivity data for groundwater exploration. Eötvös 100th Anniversary, University of Miskolc, Miskolc, Hungary May 31, 2019.

2- Gravity data interpretation for the purpose of structural mapping. EÖTVÖS LORÁND SZAKMAI ÖRÖKSÉGE A MISKOLCI EGYETEMEN, University of Miskolc, Miskolc, Hungary, September 5, 2019.

3- Magnetic data processing using 2D inversion-based Fourier transformation algorithm. Inverziós Ankét online, University of Miskolc, Miskolc, Hungary, November 2-3, 2020.

4- Integration of potential field methods to delineate the subsurface structural pattern. Youth Performers Day 2021 online, University of Miskolc, Miskolc, Hungary, May 25, 2021.

5- Noise reduction in gravitational data utilizing a 2D inversion-based Fourier transformation method. Doktoranduszok Fóruma 2021 Online, University of Miskolc, Miskolc, Hungary, November 18, 2021.

6- Processing of regularly and randomly sampled incomplete datasets using inversion-based 1D and 2D Fourier transformation. Youth Performers Day 2022, University of Miskolc, Miskolc, Hungary, May 25, 2022.