MIKOVINY SÁMUEL DOCTORAL SCHOOL OF EARTH SCIENCES

Theses of doctoral dissertation

INVERSION-BASED FOURIER TRANSFORMATION ALGORITHM USED IN PROCESSING NON-EQUIDISTANTLY MEASURED MAGNETIC DATA.

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I. SCIENTIFIC BACKGROUND AND AIMS

Data processing is an essential discipline in the science and engineering fields of study. The ability to acquire quality information from interpretation largely depends on the efficacy of the data processing method applied. In geophysics applications where interpretations are made from data collected at the earth's surface to forecast subsurface features, the quality of the processing method is of great importance. In a broader perspective, this thesis focuses on the development of new methods in inversion-based Fourier transformation for geophysical applications in the area of regular and random data processing.

The continual improvement in geophysical data acquisition over the years require more advanced data processing methods. Data translation from a time domain to frequency domain is commonly practiced in geophysical data processing, which enhances interpretation, especially in signal processing. This change can be realized through the application of Fourier transformation. For individually sampled time-domain datasets, the Discrete Fourier Transformation (DFT) algorithm is usually applied to determine its Discrete Frequency Component (DFC). As measured data often contain noise, the noise sensitivity of the processing methods is an essential feature. The noise recorded in the time domain is directly transformed into the frequency domain. Hence, the traditional discrete variants of Fourier transformation, although very stable, are noise sensitive techniques that require improvement.

To reduce this problem, Dobroka et al, 2015 formulated the Fourier transformation as an overdetermined inverse problem using Hermite functions as basis functions of discretization (the H-LSQ-FT and H-IRLS-FT) which proved to have a higher noise reduction capability even in the face of randomly occurring outliers than the traditional Discrete Fourier Transform (DFT). The method was generalized to 2D, and an application was presented in solving a reduction to the pole of a magnetic data set (Dobróka et al., 2017). Geophysical data processing covering inverse problem theory has a collection of methods with outstanding noise rejection capabilities. This necessitated the proposition to handle 1D Fourier Transform as an overdetermined inverse problem (Dobroka et al. 2015). As established in the inverse problem theory, the simple least-squares give the best solution only when data noise follows Gaussian distribution. For outliers that are irregularly distributed large errors, the estimated model parameters may be highly inconclusive, which constitute a restrictive factor to the application of the least-squares method since geophysical measurements routinely contain outliers. To achieve statistical robustness, various methods have been developed over the years to deal with data outliers.

A commonly applied robust optimization method, the Least Absolute Deviation (LAD), minimizes the L_{1}-norm characterizing the misfit between the observed and predicted data, and can be numerically achieved by using the Iteratively Reweighted Least Squares method (Scales et al., 1988) or applying linear programming. Although largely used, continual practice demonstrates that inversion with minimization of the L_{1}-norm gives more reliable estimates only when a smaller number of large errors contaminate the data. An alternative solution involves the use of the Cauchy criterion, which adopts a Cauchy-distributed data noise. The IRLS procedure, which iteratively recalculates the so-called Cauchy weights, results in a very efficient robust inversion method (Amundsen et al. 1991). The application of data weights in inversion is very crucial to guarantee each data contribute to the solution based on its error margin. Cauchy inversion is normally applied in geophysical inversion as a robust optimization method. The integration of the IRLS algorithm with Cauchy weights, though a useful procedure, problematic since the scale parameter of the weights has to be known prior to the inversion. Steiner (1988,1997) adequately solved this challenge by deriving the scale parameters from the real statistics of the data set in the framework of the Most Frequent Value (MFV) method. Dobróka et al. 1991 established globally that the MFV-weights calculated on the bases of Steiner's method result in a very efficient robust inversion method by inserting them into an IRLS procedure.

In the algorithm development of the H-LSQ-FT and the H-IRLS-FT, the discretization of the Fourier spectrum required the numerical scaling of the Hermite functions used as basis functions of discretization. Unfortunately, the values of the scaling parameters were inserted manually into the H-LSQ-FT and H-IRLS-FT algorithms from practical experience based on one's own discretion, hence, problematic. The aims of this study was to develop a new inversion-based Fourier transformation method using a different basis functions of discretization (the Legendre polynomials) thereby eliminating the scaling parameter component of the algorithm associated with the use of scaled Hermite functions. Also, I envisaged a further improvement in the existing H-LSQ-FT algorithms by introducing a meta-algorithm to optimize the scale parameter thereby eliminating the human component. Practically, my objective was also to demonstrate the applicability of the series expansion based inverse Fourier transform in processing non-equidistantly (randomly) acquired geophysical data.

II. ACCOMPLISHED INVESTIGATION

In my PhD thesis, I developed a new 1D and 2D series expansion-based inversion Fourier transformation method using Legendre polynomials as basis functions of discretization. The method treats the Fourier transformation as an overdetermined inverse problem. The spectrum is discretized by series expansion using Legendre polynomials as basis function and the inversion problem is solved for the series expansion coefficients by the LSQ and IRLS methods using Steiner weights. The new methods and the traditional DFT were tested numerically using a synthetic Morlet signal and a 2D test surface in the presence of both Gaussian and Cauchy noise. The results fully demonstrate the reduced outlier and random noise sensitivity of the newly developed algorithm compared to the traditional DFT method.

To improve the H-LSQ-FT Method, I investigated the possibility of optimizing the scale parameter using the Simulated Annealing (SA) Method, which is a Global Optimization Technique. An objective function was generated in the form of an energy function to be optimized. The SA program randomly creates a scaling frequency $'f_0'$ and energy $'Ene\_min'$ to be minimized in each step of the iteration. The inversion based Fourier transform is called into the algorithm in each iteration to re-calculate the energy function for the cycle to continues. The algorithm accepted a new solution (including the $'f_0$ scaling frequency and the alpha parameter) if the estimated energy is less than that in the previous step of iteration, otherwise the acceptance is based on the Metropolis acceptance condition. The input temperature declines continually for each iteration until the stop criteria is met and the algorithm proceeds to the optimal solution. With this procedure, the alpha parameter was optimized from the real statistics of the input data or output spectrum or a combination of both, hence, eliminating the human error component associated with defining the alpha parameter. I numerically tested the meta-algorithm in the noiseless and noisy case. With the above alpha optimizing algorithm, it is certain that the H-LSQ-FT Method will exhibit a significant improvement in its noise reduction capabilities.

With the advancement in survey equipment's which incorporate the Global Positioning System, random field survey has been made possible. I tested the H-LSQ-FT and the 2D H-LSQ-FT on synthetic and field data sampled at equidistant and non-equidistant intervals. The output maps showed the efficiency of the inversion-based Fourier Transformation method in processing randomly acquired data. This new feature will go a long way to ease geophysical field survey as field measurement are not necessary to be taken on equal grid, reducing the ensuing cost of geodetic related works in planning a geophysical field survey. In this thesis, it has been adequately demonstrated that the inversion-based H-LSQ and H-IRLS Fourier transformation algorithm can be effectively used in processing data set collected in non-equidistant (random walk) measurement geometry synthetic and field data.

III. NEW SCIENTIFIC RESULTS

Thesis 1

I developed a new 1D Legendre polynomial based Fourier transformation method in which the Jacobi matrix is calculated as an inverse DFT of the basis functions.

A) I formulated the L-LSQ-FT algorithm in which the L2-norm of the deviation vector between the observed and the calculated data was minimized using the Gaussian least squares (LSQ) method.

B) I formulated the L-IRLS-FT algorithm by minimizing the weighted norm of the deviation vector between the observed and the calculated data using Cauchy-Steiner weight. The method was robustified using the Iteratively Reweighted Least Squares method.

C) I tested the new methods for noise reduction using synthetic data. By comparing them to the traditional DFT method, it was established that the L-LSQ-FT and the L-IRLS-FT methods have a higher noise reduction capability, the later has sufficient additional outlier rejection ability.

Thesis 2

I developed a new 2D Legendre polynomial based Fourier transformation method in which the Jacobi matrix is calculated as a 2D inverse DFT of the basis functions.

A) I formulated the 2D L-LSQ-FT algorithm in which the L2-norm of the deviation vector between the observed and the calculated data was minimized using the Least Squares (2D LSQ) Method

B) I formulated the 2D L-IRLS-FT algorithm by minimizing the weighted norm of the deviation vector between the observed and the calculated data using Cauchy-Steiner weight. The method was robustified using the 2D Iteratively Reweighted Least Squares (IRLS) method.

C) I tested the new series expansion based FT methods for noise reduction using 2D synthetic data. By comparing the new methods to the traditional 2D DFT method, it was established that the 2D L-LSQ-FT and the 2D L-IRLS-FT methods have a higher noise reduction capability. I showed that the 2D L-IRLS-FT method has sufficient outlier rejection capacity.

Thesis 3

I developed a meta algorithm to optimized the scale parameter value in the H-LSQ-FT algorithm in which the optimization was calculated based on the distances defined in

A) Data space:

$$Ene = \sum_{k=1}^{N} (d^{measured} - d^{calculated})^2 = Min$$

B) Spectrum or model space:

$$Ene = \sum_{k=1}^{N} (|U_n^{cal}(\omega) - U_n^{DFT}(\omega)|)^2 = Min$$

C) Combined data and spectrum space:

$$Ene = \sum_{k=1}^{N} (d^{measured} - d^{calculated})^2 + \sum_{k=1}^{N} (|U_n^{cal}(\omega) - U_n^{DFT}(\omega)|)^2 = Min$$

I proved that for practical purposes, optimization in the data space is more efficient.

Thesis 4

I tested the efficiency of the series expansion based Fourier transformation methods (H-LSQ-FT and the H-IRLS-FT) in processing equidistant and non-equidistant (Radom-walk) geometry synthetic data sets in 1D and 2D dimension using

A) A Morlet signal
B) A magnetic dipole sampled at Equidistant intervals
C) A magnetic dipole sampled at Non-Equidistant Intervals
D) A magnetic data sampled at both equidistant and non-equidistant intervals

It was proved that the series expansion based Fourier transformation method (H-LSQ-FT and the H-IRLS-FT) can be used to adequately process dataset in both regularly sampled and random-walk (non-equidistantly sampled) intervals.

Thesis 5

I demonstrated the efficiency of the H-LSQ-FT method in processing randomly sampled data using real field magnetic measurements. I proved that the H-LSQ-FT method gives the same results in processing field magnetic data sampled at equidistant and non-equidistant grid.

PRACTICAL APPLICATION OF THE RESULTS

In frame of my dissertation I performed geophysical inversion method development for data processing. The developed algorithms can be considered as an advanced and efficient methods in the inversion based Fourier transformation. These methods have sufficient noise suppression capability and outlier resistance, therefore there is a possibilities of applications in the geophysical data processing and interpretation as well as other areas where Fourier transform is applied for the processing of noisy data. It is known that the Fourier transform is applied in various fields of technical and nature sciences, therefore all sub-areas where the data processing of noisy data is relevant (for example image processing and remote sensing) can be considered a potential application field of the inversion based Fourier transform methods.

A very significant advantage of the inversion based Fourier transform methods is that they give a good results irrespective of the sampling protocol applied in the field of survey, in that, it give the same output result for equidistant and non-equidistant measurement. This important new findings can be applied in processing data acquired in others scientific areas where random measurements are of great value such as astronomical studies.

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IV. LIST OF RELATED PUBLICATIONS AND PRESENTATIONS

JOURNAL ARTICLES

1. Daniel O. B. Nuamah and Mihály Dobróka (2018). Reduction to pole of non-equidistantly measured magnetic data using an inversion-based Fourier transformation algorithm. Geosceince and Engineering Journal, Volume 6, Number 9 (2018), pg 32-39. HU ISSN 2063-6997

2. Nuamah, D.O.B. & Dobroka, M. (2019). Inversion-based fourier transformation used in processing non-equidistantly measured magnetic data. Acta Geod Geophys (2019). https://doi.org/10.1007/s40328-019-00266-4

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1. A. Kiss, M. Dobróka, D.O.B. Nuamah and J. Somogyi Molnár (2018). Noise Suppression in Reduction to Pole of Magnetic Data. EAGE Saint Petersburg 2018 Conference (Innovation in Geoscience-Time for Breakthrough, Saint Petersburg, Russia, 09-12 April 2018) DOI: 10.3997/2214-4609.201800204

2. A. Kiss, D.O.B. Nuamah and M. Dobróka (2018). Improved Description of the Acoustic Wave Parameters – Pressure Relations with New Double Relaxation Models, EAGE Near Surface Geoscience Conference (24th European Meeting of Environmental and Engineering Geophysics, Porto-Portugal, 09th -13th September 2018), DOI: 10.3997/2214-4609.201802583

3. Nuamah Daniel O.B., A. Kiss and M. Dobroka (2018). Reduction to Pole of Non-Equidistantly Measured Magnetic Data Using an Inversion-Based Fourier Transformation Algorithm, EAGE Near Surface Geoscience Conference, (2nd Conference on Geophysics for Mineral Exploration and Mining, Porto-Portugal, 09th - 13th September 2018), DOI: 10.3997/2214-4609.201802725

4. Mihaly Dobroka, Daniel O.B Nuamah, J. Somogyi-Molnar and Tamas Ormos (2018), A robust inversion-based Fourier Transformation algorithm used in the interpretation of non-equidistantly measured magnetic data, Proceedings at the 9th ICCM2018, 6th-10th August 2018, Rome, Italy, Eds: G.R. Liu, Patrizia Trovalusci, ID 3569, ScienTech Publisher. ISSN 2374-3948

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1. Nuamah Daniel O.B., A. Kiss and M. Dobroka (2018). Reduction to Pole of Non-Equidistantly Measured Magnetic Data Using an Inversion-Based Fourier Transformation Algorithm, EAGE Near Surface Geoscience Conference, (2nd Conference on Geophysics for Mineral Exploration and Mining, Porto-Portugal, 09th - 13th September 2018), DOI: 10.3997/2214-4609.201802725

2. Mihaly Dobroka, Daniel O.B Nuamah, J. Somogyi-Molnar and Tamas Ormos (2018), A robust inversion-based Fourier Transformation algorithm used in the interpretation of non-equidistantly measured magnetic data, Proceedings at the 9th ICCM2018, 6th-10th August 2018, Rome, Italy, Eds: G.R. Liu, Patrizia Trovalusci, ID 3569, ScienTech Publisher. ISSN 2374-3948

DOMESTIC CONFERENCES PRESENTATION

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2. Nuamah Daniel Oduro Boatey (2018). The Use of Inversion-Based Fourier Transformation Algorithm in Processing Non-Equidistantly Measured Magnetic Data. Inversion Conference-Miskolc, 21st April,2018

3. Nuamah Daniel Oduro Boatey (2018). The Use of Inversion-Based Fourier Transformation Algorithm in Processing Non-Equidistantly Measured Magnetic Data. PhD Forum, Faculty of Earth Science Engineering Session, University of Miskolc, 22nd November,2018