MIKOVINY SÁMUEL DOCTORAL SCHOOL OF EARTH SCIENCES

Theses of doctoral dissertation

INVERSION METHOD DEVELOPMENT FOR REDUCING NOISE SENSITIVITY OF FOURIER TRANSFORM

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I. SCIENTIFIC BACKGROUND AND AIMS

The Fourier transform is one of the most important data processing operations on the field of technical and nature sciences, however it is the starting point for numerous other applications as well. In the engineering practice the discrete Fourier transform (DFT) and fast Fourier transform (FFT) are the tools for determining the frequency spectrum in case of discrete time domain datasets. These algorithms approximate the spectrum of the time signal with sufficiently high accuracy, if the sampling and the registration time ranges are set properly. However, the measured data are always contaminated by noise, it is therefore particularly important to study the noise reduction capability of the applied operations. The Fourier transform - in its traditional form - is highly noise sensitive, therefore it is often problematic to process the measurement data.

The basic task of the geophysical inversion is to read out the geological information from the measured data and to define the petrophysical and geometrical parameters of geological structures. In the field of geophysical inversion several methods exist to suppress the noise very well. The researchers at the Department of Geophysics, University of Miskolc worked out several procedures during the development of inversion methods (Vass and Dobróka (2010), Dobróka and Völgyesi (2010), Gyulai et al. (2010), Gyulai and Ormos (1999), Kis (1998, 2002), Turai et al. (2010), Turai (1981), Dobróka and Szabó (2010), Szabó (2004)), which obtained internationally accepted success in the field of noise suppression.

The main purpose of the inversion researches at our Department is the processing and evaluation of data measured on complex (laterally and vertically inhomogeneous) geological structures by using series expansion discretization where the expansion coefficients are defined in an inversion process. The main advantage of this method is that appropriate resolution can be achieved by introducing relatively small number of expansion coefficients so that the task leads to over-determined inverse problem. The idea of series expansion based inversion was used in various fields of geophysics. In the inversion of borehole geophysical data (Dobróka et al. 2010) the depth dependent physical parameters were written as series expansion; the series expansion coefficients were defined within the framework of the inversion process. A new method were presented for the processing of induced polarization (IP) data using the series expansion inversion by Turai et al. (2010). Based on Eötvös torsion and gravity measurements, deflections of the vertical and digital terrain model data the inversion based reconstruction of the three-dimensional gravity potential was presented by Dobróka and Völgyesi (2010). The Fourier transform was handled as a series expansion based inverse problem by Dobróka and Vass (2010). An efficient method for the series expansion based inversion of geoelectric data measured on two-dimensional geological structures was shown (Gyulai et al. 2010).

In my research work, following Vass (2010) (who investigated the use of Hermite functions, interval constant functions and Dirac-delta functions as basis functions in the inversion based Fourier transformation) I developed a new, and consistent mathematical formalism. The aim of my research work was also to increase the robustness of the new inversion based Fourier transform algorithms, i.e. they work suitably in case of datasets containing outliers. The numerical investigation results showed efficient noise reduction capability (Szegedi and Dobróka 2012, Dobróka et al. 2012).

II. ACCOMPLISHED INVESTIGATIONS

In my dissertation I discussed in detail the new algorithm of the inversion based Fourier transform following the series expansion based inversion methodology developed at the Department of Geophysics, University of Miskolc. In this approach it follows that the elements of the Jacobi-matrix are given as the inverse Fourier transform of the basis functions. From this fact there arises the idea for choosing the basis functions from the eigenfunctions of the inverse Fourier transform, because in this case the elements of the Jacobi-matrix can be calculated by means a simple explicit formula. Following the procedure published by Vaidyanathan (2008) I showed that the applied generating function $e^{-\omega^2/2}$ leads to the Hermite function (except the normalizing factor). Therefore they can be chosen as basis functions of the series expansion. Vass (2010) noted that the scaled Hermite functions can be treated in a more flexible way in series expansion inversion, so they were chosen as basis functions. I gave the transformation formula between the unscaled and scaled Hermite functions and by using them I showed that the Jacobi-matrix can be directly expressed with the unscaled Hermite function. Thereby I obtained a simple formula regarded as the linear combination of series expansions for the determination of theoretical data in the forward problem. With its application I introduced the LSQ-FT method with minimizing the L_2-norm of the deviation vector and treated the Fourier transform as purely over-determined inverse problem. I developed the algorithms of the methods, wrote softwares in Matlab and illustrated the operation of these methods on numerical tests. I compared the spectra of Gaussian noisy datasets computed by DFT and LSQ-FT and showed that the latter method is less sensitive to noise.

Investigations of the LSQ-FT inversion algorithm which is based on the scaled Hermite function discretization confirmed on datasets including Gaussian and Cauchy noise that it is necessary to develope robust/resistant methods, which behave more resistive against the noises of measured data. Many practical examples demonstrated that applying the Most Frequent Value (MFV) method (Steiner 1988), the geophysical inversion methods become more robust. Thus, the new robust Fourier transform algorithm (S-IRLS-FT) including the Steiner-weights defined in the framework of the MFV method and can be solved with the Iteratively Reweighted Least Squares method. In this new algorithm the Steiner-weights are computed from the measured data set in an internal iteration procedure. Numerical studies have been demonstrated the effectiveness of this method (both in the frequency and the time interval a significant improvement was found). I computed the spectra of the data set contaminated by noise following Cauchy distribution by means of DFT and the newly developed S-IRLS-FT and compared the results. The test confirmed the efficiency of the S-IRLS-FT method and demonstrated its sufficient noise reduction capacity.

Furthermore, generalizing the 1D algorithm I introduced the inversion based two dimensional Fourier-transform algorithm in its LSQ (2D-LSQ-FT) and IRLS (using Steiner's weights) versions (2D-S-IRLS-FT). The resistance against outliers and the noise suppression capabilities of the S-IRLS-FT made it justifiable to try the method on other data processing and earth science fields. That is why it was tested in seismic data processing where double inversion was used. Firstly, the Fourier spectrum of the time signal was defined by inversion based method, secondly (after the modifications necessary for the Hilbert transform) we returned to the time domain by robust inversion. I discussed the theoretic background of the developed Hilbert transform method and the results of the studies. I performed numerical tests on Gaussian and Cauchy noisy datasets, the results showed clearly that the algorithm provides an order of magnitude better noise suppression capability compared to the conventional method (based on the traditional DFT), (Szegedi and Dobróka 2014). In the field of the reduction to pole of surface measured magnetic data I studied the efficiency of the two dimensional algorithm (2D-S-IRLS-FT), and here I also verified sufficient improvements compared to the traditional methods (using 2D-DFT).

III. NEW SCIENTIFIC RESULTS

Thesis 1

In case of series expansion based discretization the Jacobi-matrix of the inverse problem is the inverse Fourier transform of the basis functions. My aim was to apply basis functions, which are eigenfunctions of the Fourier transform because in this case the elements of the Jacobi-matrix can easily be defined as multiplication of the eigenvalue and the basis function. Following Vaidyanathan (2008) method I demonstrated that the unscaled Hermite functions are the eigenfunctions of the Fourier transform, which can be applied advantageously in developing inversion based Fourier transform algorithm with series expansion discretization.

a) Accordingly, I gave the necessary formulas for the Jacobi-matrix and the forward problem, applying the unscaled and the scaled Hermite function.

b.) Using with new calculation method, in computing the L_2-norm of deviation vector of measured and calculated data, I modified the H-LSQ-FT method which was introduced by Vass (2010). I tested the new LSQ inversion method (LSQ-FT) discretized by the scaled Hermite function.

Thesis 2

Minimizing the weighted norm of deviation vector between measured and calculated data I introduced the robust inversion based Fourier transform method which was discretized by the scaled Hermite function (S-IRLS-FT).

a) During the method I worked with the weighted matrix elements (obtained by the Most Frequent Value method) because the ε^2 scale parameter can be derived directly from the measured dataset in an internal iteration procedure.

b) Testing the effectiveness of S-IRLS-FT method on Cauchy noisy dataset I found that in case of Cauchy distribution noise it provides excellent results. The new method has remarkable noise reduction capability compared to the conventional DFT method.

Thesis 3

Using the scaled Hermite functions for the discretization of frequency spectrum I developed 2D inversion based Fourier transform method. I gave an explicit formula for the Jacobi-matrix and with this obtained a fast, very simple linear formula for the solution of the forward problem.

a) I introduced the 2D-LSQ-FT inversion based Fourier transform method by minimizing according to L_2-norm of deviation vector of the measured and calculated data. During the method the discretization occurred by scaled Hermite functions.

b) Minimizing the weighted norm of the deviation vector between the measured and calculated data I introduced the 2D-S-IRLS-FT inversion based robust Fourier transform method. It was based on discretization by scaled Hermite functions and the Steiner-weights were applied in the definition of weighted norm.

Thesis 4

I gave a new robust inversion method for preparation of the Hilbert transform based on the one-dimensional Fourier transform method. During the process for calculation of the Fourier transform the S-IRLS-FT robust inversion method was applied. After taking into account the transfer function of Hilbert transform, while returning to the time domain I introduced the robust inversion based algorithm for the one-dimensional inverse Fourier transform.

a) With the new robust Hilbert transform it is possible to generate the analytic signal and the robust seismic attributes.

b) I applied the new method for calculation of the reflection intensity (first attribute) and testing on Gaussian and Cauchy noisy datasets. Based on the inversion results it can be said that the algorithm provides an order of magnitude better noise suppression capability compared to the conventional (calculated by DFT) section.

Thesis 5

In the field of pole reduction of surface measured magnetic data the 2D-S-IRLS-FT Fourier transform was used. Based on the numerical testing I observed that the method has efficient noise reduction capability (compared to the conventional DFT method). This robust 2D Fourier transform method has outstanding ability to improve the accuracy of the pole reduction.

PRACTICAL APPLICATION OF THE RESULTS

In frame of my dissertation I performed geophysical inversion method development. The developed algorithms can be considered as novel methods in the inversion based Fourier transformation. These methods have sufficient noise suppression capability and outlier resistance, therefore they carry on the possibilities of applications in the geophysical data processing and interpretation and so all areas, where Fourier transform is applied for the processing of noisy data (can be listed here the examined Hilbert transform and the 2D magnetic data pole reduction). Significant advantage of the developed Fourier transform methods that they does not require equidistant (regular) measurement system.

It is known that the Fourier transform is applied in various fields of technical and nature sciences, therefore all sub-areas where the data processing of noisy data is relevant (for example image processing and remote sensing) can be considered a potential application field of the inversion based Fourier transform methods discussed in the Thesis.

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