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5 Conclusions

Any point in the Euclid plane corresponds to an elliptic complex number in nature, which combines the x and y coordinates as a whole. Based on the consideration, the planar coordinate transformation model in the complex field is constructed. The model is more concise than the conventional transformation model in the real number field. In the case of complex transformation model, the conventional least squares is invalid. To estimate the complex transformation parameter, the contribution introduces the complex least squares and presents the complex Gauss-Jacobi combinatorial algorithm. The two approaches are employed to a simulative and an actual case study and the result indicate that the complex least squares is a correct and efficient least squares in the complex number field, and can obtain the identical estimation as the least squares for the real transformation model. Unfortunately, the complex least squares is not immune from the gross error as the conventional real least squares. In essence, the conventional real least squares is a special case of the complex least squares, which can be easily found by comparing Eq. (18) and (47), and Eq. (47) is capable of extending to the form considering the weight matrix of observation vector. On the contrary, the complex Gauss-Jacobi combinatorial algorithm is verified as a good robust estimation.

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On the use of Steiner's weights in inversion-based Fourier transformation: robustification of a previously published algorithm

H. Szegedi · M. Dobróka

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Abstract In our previous paper (Dobróka et al. *Acta Geod Geophys Hung* 47(2):185–196, 2012) we proposed a new robust algorithm for the inversion-based Fourier transformation. It was presented that the Fourier transform and its variants responds very sensitively to any little measurement noise affected an input data set. The continuous Fourier spectra are assumed as a series expansion with the scaled Hermite functions. The expansion coefficients are determined by solving an over-determined inverse problem. Here, we use the new Steiner's weights (previously called the weights of most frequent values or abbreviated as MFV), where the scale parameter can be determined in an internal iteration process. This method results a very efficient robust inversion method in which we calculate the Steiner weights from iteration to iteration into an IRLS procedure. The new method using the Steiner's weights is also numerically tested by using synthetic data.

Keywords Inversion-based Fourier transformation · IRLS method · Cauchy noise · Steiner weights · Noise reduction capability

1 Introduction

In geophysical interpretation it is always an important task to reduce the influence of data noises. To do this in the field of geophysical inversion various methods has been developed. It is well-known from inverse problem theory that simple least square methods give optimal results only when data noises follow Gaussian distribution. The practice of geophysical inversion shows that the least square solutions are very sensitive to sparsely distributed large errors, i.e. outliers in the data set and the estimated model parameters may even be completely

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non-physical. More generally, the distribution of the errors in the measured data is seldom Gaussian so that the use of the least squares method (LSQ) cannot be optimal.

There are various ways to address the question of the statistical robustness. One of the most frequently used methods in robust optimization is the least absolute deviation (LAD) method. In this case L_1 norm is used to characterize the misfit between the observed and the predicted data. The inversion with minimization of the L_1 norm gives more reliable estimates when a small number of large errors contaminate the data. LAD inversion can be numerically realized by using linear programming or (after Scales et al. 1988) applying iteratively reweighted least squares method (IRLS). Another possibility is the use of the Cauchy criterion. In this case the Cauchy distribution of data noise is assumed. Cauchy inversion is also frequently used in the geophysical inversion as a robust optimization method (Amundsen 1991). The IRLS algorithm involving Cauchy weights is a very useful procedure, but it has got a problem that the scale parameter is to be a priori given. This difficulty was elegantly eliminated by Steiner (1988) who derived the scale parameters from the real statistics of the data set in the framework of the most frequent values method (MFV). It was first proved in the international literature by Dobróka et al. (1991) that the MFV-weights calculated on the base of Steiner's method results in a very efficient robust inversion method by inserting them into an IRLS procedure.

In a previous paper (Dobróka et al. 2012) we published an improved algorithm for treating the Fourier transformation as inverse problem. It was shown that the continuous Fourier transform and its variants the discrete Fourier transform (DFT) and the fast Fourier transform (FFT) algorithms respond very sensitively to any little measurement noise affected the input data set. To address this problem we formulated the Fourier transformation as an overdetermined inverse problem. An essential step of this approach is the use of a special discretization of the real and imaginary part of the spectrum. Following a new inversion strategy developed at the Geophysical Department of the University of Miskolc as discretization tool we used series expansion. As basis functions we used Hermite functions which gave us the advantage that the elements of the Jacobi's matrix can be calculated by means a simple explicit formula.

In this paper we further develop the previously published algorithm (Dobróka et al. 2012) by inserting Steiner weights (instead of Cauchy ones) into the IRLS based inversion Fourier transform method.

2 Theoretical background

For the one-dimensional continuous Fourier transform we use the formula

$$U(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(t) e^{-j\omega t} dt, \quad (1)$$

where t denotes time, ω is the angular frequency and j is the imaginary unit. The frequency spectrum $U(\omega)$ is the Fourier transform of a real valued time function $u(t)$ and it is generally a complex valued continuous function. By the means of the inverse Fourier transform formulated hereunder

$$u(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(\omega) e^{j\omega t} d\omega, \quad (2)$$

we can return from the frequency domain to the time domain. In defining the Fourier transform as an inverse problem, the frequency spectrum $U(\omega)$ should be described by a discrete parametric model. In order to satisfy this expectation, let us assume that $U(\omega)$ is approximated

with sufficient accuracy by using a finite series expansion

$$U(\omega) = \sum_{i=1}^M B_i \Psi_i(\omega), \quad (3)$$

where the parameter B_i is a complex valued expansion coefficient and Ψ_i is a member of an accordingly chosen set of real valued basis functions.

Using the terminology of (discrete) inverse problem theory, the theoretical values of time domain data can be given by the inverse Fourier transform

$$u_k^{theor}(t_k) = u_k^{theor} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(\omega) e^{j\omega t_k} d\omega, \quad (4)$$

where t_k is the k -th sampling time. Inserting the expression given in Eq. (3) one finds that

$$u_k^{theor} \cong \int_{-\infty}^{\infty} \left(\sum_{i=1}^M B_i \Psi_i(\omega) \right) e^{j\omega t_k} d\omega = \sum_{i=1}^M B_i \int_{-\infty}^{\infty} \Psi_i(\omega) e^{j\omega t_k} d\omega. \quad (5)$$

Let us introduce the notation

$$G_{k,i} = \int_{-\infty}^{\infty} \Psi_i(\omega) e^{j\omega t_k} d\omega, \quad (6)$$

where $G_{k,i}$ is an element of the Jacobi's matrix of the size (N is the number of time domain data and M is the number of unknown expansion coefficients). It can be noted, that the Jacobi's matrix is the inverse Fourier transform (in $t = t_k$) of the Ψ_i basis function.

After this step, the theoretical values can be written in the linear form

$$u_k^{theor} = \sum_{i=1}^M B_i G_{k,i}. \quad (7)$$

The parametrization of a model is always an important step in constructing an inversion algorithm. As a frequency spectrum is defined over the interval $(-\infty, \infty)$ the choice of a set of basis functions defined over the same interval would be advantageous. In addition, the use of a set of orthonormal functions for the series expansion is generally prosperous to the parametrization of the model. In order to fulfil these requirements, we have chosen the set of scaled Hermite functions, especially because they are square-integrable (which ensures the existence of their Fourier transform).

Following the algebra presented in our previous work (Dobróka et al. 2012) let us consider the basic formula of the scaled Hermite polynomials and Hermite functions. The Rodriguez formula for scaled Hermite polynomials takes the form

$$h_n(\omega, \alpha) = (-1)^n e^{\alpha\omega^2} \left(\frac{d}{d\omega} \right)^n e^{-\alpha\omega^2} \quad (8)$$

and can be generated by the recursion formula

$$h_{n+1}(\omega, \alpha) = 2\omega\alpha h_n(\omega, \alpha) - 2n\alpha h_{n-1}(\omega, \alpha), \quad (9)$$

where α is the scale factor and $h_0(\omega, \alpha) = 1$, $h_1(\omega, \alpha) = 2\alpha\omega$ (Gröbner and Hoffreiter 1958). The normalizing equation is

$$\int_{-\infty}^{\infty} e^{-\frac{\alpha\omega^2}{2}} \cdot h_n(\omega, \alpha) \cdot h_m(\omega, \alpha) d\omega = \sqrt{\frac{\pi}{\alpha}} (2\alpha)^n n! \delta_{nm}, \quad \delta_{nm} = \begin{cases} 0, & n \neq m \\ 1, & n = m \end{cases} \quad (10)$$

and the scaled Hermite functions can be defined as

$$H_n(\omega, \alpha) = \frac{e^{-\frac{\alpha\omega^2}{2}} \cdot h_n(\omega, \alpha)}{\sqrt{\sqrt{\frac{\pi}{\alpha}} n! (2\alpha)^n}}. \quad (11)$$

The normalizing equation in case of the modified (scaled) Hermite function is

$$\int_{-\infty}^{\infty} H_n(\omega, \alpha) \cdot H_m(\omega, \alpha) d\omega = \delta_{nm}, \quad \delta_{nm} = \begin{cases} 0, & n \neq m \\ 1, & n = m \end{cases}. \quad (12)$$

Expanding the spectrum by means of the modified Hermite functions, in accordance with Eq. (6) the elements of the Jacobi's matrix represent the inverse Fourier transforms of the $H_n(\omega, \alpha)$ basis functions

$$G_{kn} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_n(\omega, \alpha) \cdot e^{j\omega t} d\omega. \quad (13)$$

It was proved (Dobróka et al. 2012) that

$$G_{kn} = \frac{1}{\sqrt[4]{\alpha}} (j)^n H_n^{(0)}(t') = \frac{1}{\sqrt[4]{\alpha}} (j)^n H_n^{(0)}\left(\frac{t}{\sqrt{\alpha}}\right). \quad (14)$$

This is a very important result in further developing the inversion-based Fourier transform method because the Jacobi's matrix can be produced quickly, as the procedure do not require integration.

3 IRLS algorithm with the use of Steiner weights

In accordance with Eq. (7) the theoretical data can be calculated as a linear expression of the expansion coefficients using the easily calculated elements of the Jacobi's matrix. The general element of the deviation vector can be given in the following form

$$e_k = u_k^{\text{measured}} - u_k^{\text{theor}}. \quad (15)$$

If the measured data set contains Gaussian noise, the minimization of the L_2 norm of the deviation vector is applied. This is the case of the LSQ when

$$E_2 = \sum_{k=1}^N e_k^2 \quad (16)$$

is minimized resulting in the well-known set of the normal equations

$$\mathbf{G}^T \mathbf{G} \vec{B} = \mathbf{G}^T \vec{u}^{\text{measured}}. \quad (17)$$

By solving these normal equations we can give an estimate for the complex expansion coefficients, and both the real and imaginary parts of the LSQ estimated Fourier transform (LSQ-FT) can be calculated at any frequency by using

$$U^{\text{estimated}}(\omega) = \sum_{i=1}^M B_i^{\text{estimated}} \Psi_i(\omega). \quad (18)$$

It is well-known that the LSQ gives optimal results only in the case when the noise contaminating the data follows Gaussian distribution. In more general cases other norms of the deviation vector are used. In order to define a robust inversion algorithm, the minimization of the weighted norm

$$E_w = \sum_{k=1}^N w_k e_k^2 \quad (19)$$

with the so-called Cauchy weights

$$w_k = \frac{\sigma^2}{\sigma^2 + e_k^2} \quad (20)$$

is suggested (here σ^2 is an accordingly chosen positive number). Using this norm for the solution of inverse problems provides reliable results even if the input data sets contain outliers (Dobróka et al. 2012).

There is a problem with inversion procedures involving Cauchy weights: the scale parameter should be a-priori given. This difficulty can easily be solved by using Steiner weights (Steiner 1988). In the framework of Steiner's method (previously called the method of most frequent values or abbreviated as MFV) the scale parameter σ^2 can be determined in an internal iteration loop. In the $(j+1)$ -th step of this procedure the ε_{j+1}^2 (Steiner's scale factor called dihesion) can be calculated in the knowledge of ε_j^2 as

$$\varepsilon_{j+1}^2 = 3 \frac{\sum_{k=1}^N \frac{e_k^2}{(\varepsilon_j^2 + e_k^2)^2}}{\sum_{k=1}^N \left(\frac{1}{\varepsilon_j^2 + e_k^2} \right)^2}, \quad (21)$$

where in the 0-th step the ε_0 starting value is given as

$$\varepsilon_0 \leq \frac{\sqrt{3}}{2} (e_{\max} - e_{\min}). \quad (22)$$

It can be seen that the above procedure derives the scale parameter from the data set (deviation between measured and calculated data). The stop criterion can easily be defined by experience (for example, a fixed number of iterations). After this the Steiner weights are calculated by using the (Steiner's) scale parameter given in the last step of the internal iterations having the form

$$w_k = \frac{\varepsilon^2}{\varepsilon^2 + e_k^2}. \quad (23)$$

In the case of Steiner weights the misfit function given in Eq. (19) is non-quadratic (because e_k contains the unknown expansion coefficients) and so the inverse problem is nonlinear which can be solved again by applying the method of the IRLS (Scales et al. 1988). In the framework of this algorithm a 0-th order solution $\vec{B}^{(0)}$ is derived by using the (non-weighted) LSQ method and the weights are calculated as

$$w_k^{(0)} = \frac{\varepsilon^2}{\varepsilon^2 + (e_k^{(0)})^2} \quad (24)$$

with

$$e_k^{(0)} = u_k^{\text{measured}} - u_k^{(0)}, \quad (25)$$

where

$$u_k^{(0)} = \sum_{i=1}^M B_i^{(0)} G_{ki}. \quad (26)$$

In the first iteration the misfit function

$$E_w^{(1)} = \sum_{k=1}^N w_k^{(0)} \left(e_k^{(1)} \right)^2 \quad (27)$$

is minimized resulting in the linear set of normal equations

$$\mathbf{G}^T \mathbf{W}^{(0)} \mathbf{G} \bar{\mathbf{B}}^{(1)} = \mathbf{G}^T \mathbf{W}^{(0)} \bar{\mathbf{u}}^{\text{measured}} \quad (28)$$

of the weighted LSQ where the $\mathbf{W}^{(0)}$ weighting matrix is of the diagonal form

$$W_{kk}^{(0)} = w_k^{(0)}. \quad (29)$$

This procedure is repeated giving the typical j -th iteration step

$$\mathbf{G}^T \mathbf{W}^{(j-1)} \mathbf{G} \bar{\mathbf{B}}^{(j)} = \mathbf{G}^T \mathbf{W}^{(j-1)} \bar{\mathbf{u}}^{\text{measured}} \quad (30)$$

with the $\mathbf{W}^{(j-1)}$ weighting matrix

$$W_{kk}^{(j-1)} = w_k^{(j-1)}. \quad (31)$$

(In these steps the normal equation is linear, because the weights are always calculated in a previous step. Here we note that each step of these iterations contain an internal loop for the determination of the Steiner's scale parameter). This iteration is repeated until a proper stop criterion is met.

4 Numerical results

In order to test the noise rejection ability of the robust inversion-based Fourier transform using Steiner weights (S-IRLS-FT) we generated a data set by means of the formula

$$u(t') = e^{-\frac{t'^2}{2}} \cos(2\pi f_0 t'), \quad t' = 10(t - 0.5), \quad f_0 = 2 \text{ Hz}, \quad -1 \leq t \leq 1, \quad (32)$$

which is known as a Morlet wavelet, frequently used in seismic data processing. The discrete samples were calculated equidistantly in 401 points ranging over the time interval of $[-1, 1]$. The time domain and the frequency domain representations of the noiseless signal can be seen in Figs. 1 and 2. Instead of the angular frequency ω the frequency f was used for scaling the axis of abscissa of the Fourier spectrum (calculated by DFT) in Fig. 2.

It was demonstrated in our previous paper (Dobróka et al. 2012) that the DFT is very sensitive for noises following non-Gaussian distribution. This is especially true in case of data set contaminated by Cauchy-distributed noise. For testing the S-IRLS-FT algorithm we generated a noisy data set by adding random noise following Cauchy distribution (Fig. 3).

It is shown in Fig. 4 that the DFT algorithm gives a very noisy spectrum in this case.

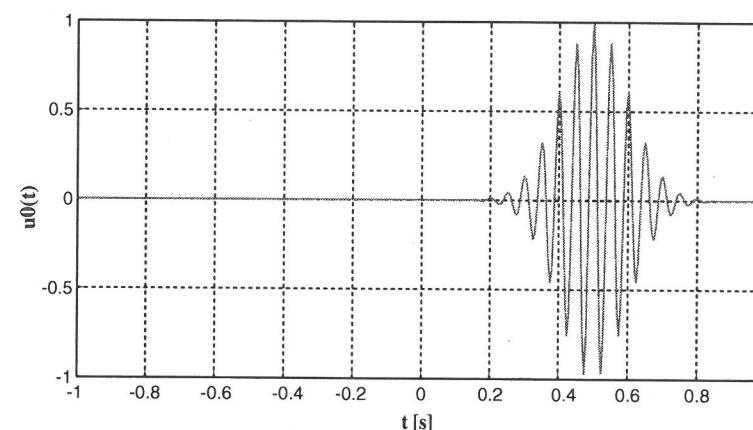


Fig. 1 The noiseless signal in the time domain

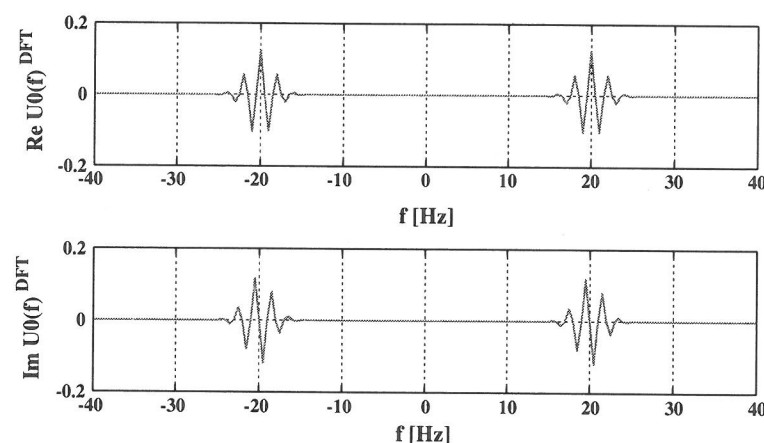


Fig. 2 The noiseless signal in the frequency domain

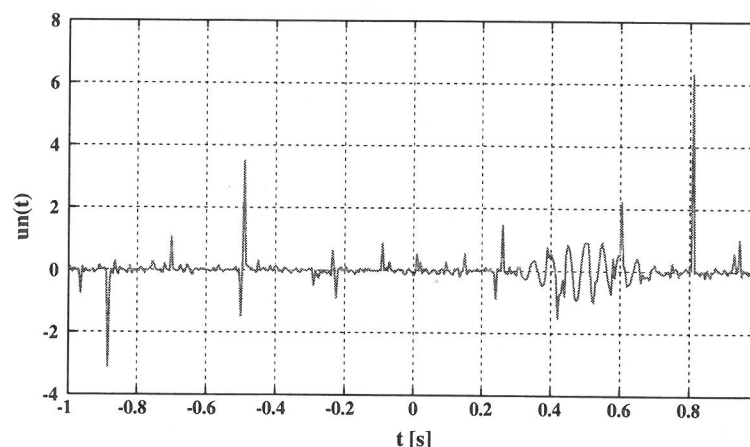


Fig. 3 The signal contaminated by Cauchy noise in the time domain

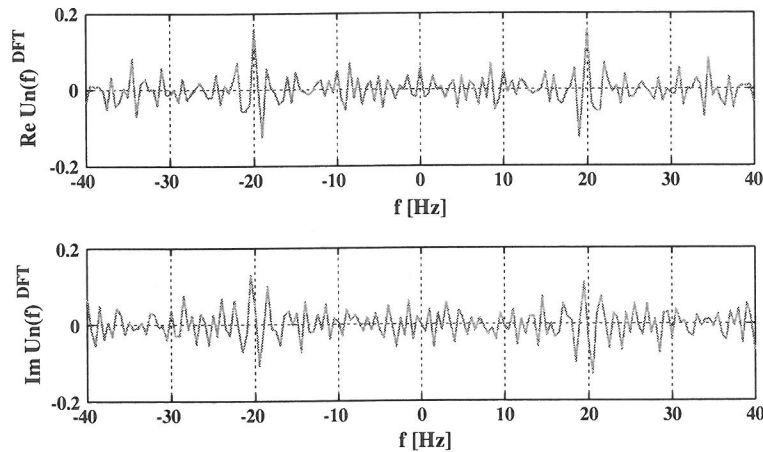


Fig. 4 The signal contaminated by Cauchy noise in the frequency domain (given by standard DFT)

In order to characterize the accuracy of the transformations we introduce the data distance

$$d = \sqrt{\frac{1}{N} \sum_{i=1}^N [u^{\text{noiseless}}(t_i) - u^{\text{calculated}}(t_i)]^2} \quad (33)$$

in the time domain and the model distance

$$D = \left[\frac{1}{N_f} \sum_{i=1}^{N_f} (\text{Re}[U^{\text{calculated}}(f_i)] - \text{Re}[U^{\text{noiseless}}(f_i)])^2 + \frac{1}{N_f} \sum_{i=1}^{N_f} (\text{Im}[U^{\text{calculated}}(f_i)] - \text{Im}[U^{\text{noiseless}}(f_i)])^2 \right]^{\frac{1}{2}} \quad (34)$$

in the frequency domain. The distance between the noisy and noiseless data sets is $d = 0.4554$. Using (34) we find the model distance between the DFT spectrum of the noisy (contaminated by Cauchy noise) and the noiseless data: $D = 0.0457$.

If we apply for the same noisy data set our S-IRLS-FT method we get an estimated spectrum shown in Fig. 5. It can be seen that compared to the DFT spectrum, this figure represents sufficient improvement, which is characterized also by the model distance between the noiseless and the noisy (given by S-IRLS-FT) data: $D = 0.0050$.

It is well known that applying inverse Fourier transform for the DFT Fourier spectrum we retrieve the noisy data set exactly. (The reason is that applying DFT or FFT we solved well-defined inhomogeneous algebraic set of equations.) By using inversion-based Fourier transform method we solve an over-determined set of equations and—for achieving sufficient noise rejection—apply Steiner weights. In this case it is important to see the time domain data set given by the inverse Fourier transform of S-IRLS-FT spectrum. The result is shown by Fig. 6. Compared to the noisy data set it can be seen that the new inversion-based Fourier transform method has appreciable noise rejection capability. This is characterized by the data distance between the noiseless and the S-IRLS-FT calculated time domain data: $d = 0.0426$. This noise rejection capability of the algorithm makes its application very promising in various fields of data processing including for example the inversion based processing of IP data (Turai 2011).

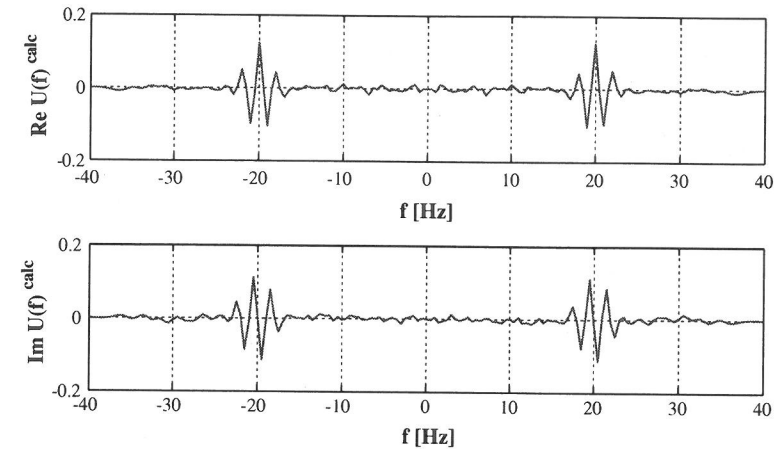


Fig. 5 The result of the new version of the S-IRLS-FT in the frequency domain

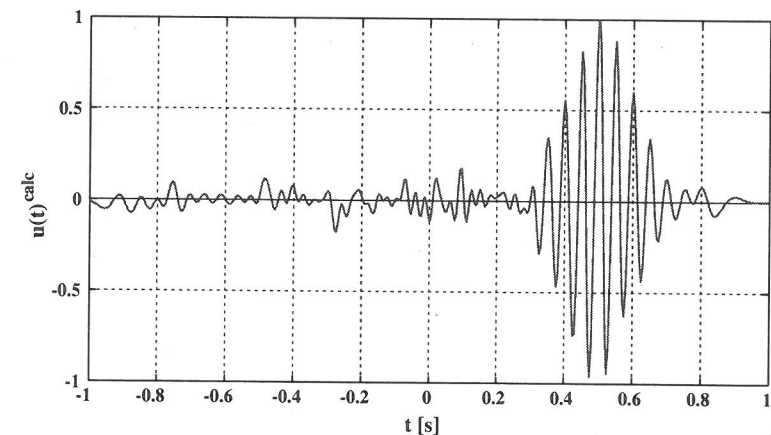


Fig. 6 The result of the new version of the S-IRLS-FT in the time domain

5 Summary

A new inversion algorithm was proposed for a more robust inversion-based Fourier transformation. For the discretization of the complex Fourier spectrum Hermite functions were applied as basis functions. In order to increase the noise rejection capability of the algorithm Steiner weights were implemented. In the framework of Steiner's most frequent value method the scale parameter of the weights are determined from the data set directly. This results in an automatic procedure for the calculation of the weights. The algorithm of the inversion-based Fourier transform was constructed using the IRLS procedure. The new S-IRLS-FT algorithm was numerically tested by using synthetic data. It was proved that compared to DFT the S-IRLS-FT method has sufficient noise rejection capability.

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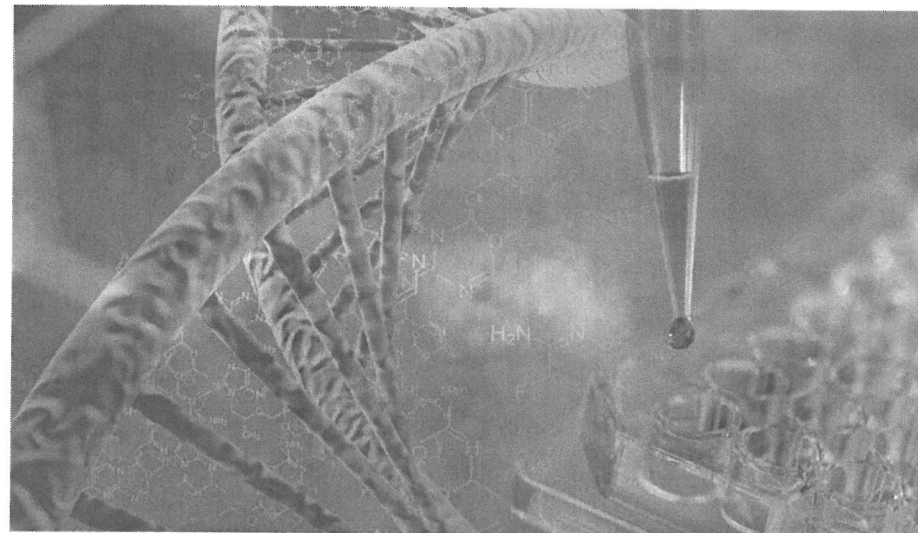
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