Modeling the acoustic hysteresis of P and S wave velocity

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Abstract

Acoustic velocity in rocks is strongly depends on pressure, indicating that wave propagation is very nonlinear and the quasistatic elastic properties of rocks are hysteretic. Characterization of hysteretic behavior is important for a mechanical understanding of reservoirs during depletion. Therefore a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which describes the connection between the propagation velocity of acoustic wave and rock pressure both in case of pressurization and depressurization cycles as well as explains the mechanism of acoustic hysteresis. The developed model is based on the idea that the pores in rocks close during pressurization and reopen during depressurization. It is valid also for S wave velocities since the basis of the model is the change of pore volume. The model was applied to acoustic P and S wave velocity data sets. The parameters of the petrophysical model were determined by a linearized inversion method. The calculated data matched accurately with measured data proving that the new rock physical model describing acoustic hysteresis applies well in practice.

Introduction

It is well known that the change of acoustic wave velocity propagating in rocks under pressure is highly nonlinear and the quasistatic elastic properties of rocks are hysteretic (Ji et al. 2007). The observable non-elastic response to pressure (acoustic hysteresis) may be caused by the processes: irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions. According to the theory of irreversible closure of microcracks, the microcracks closed during pressurization do not reopen during subsequent depressurization (Walsh and Brace 1964). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original dimensions at lower pressures (Birch 1960). By idea of the improvement of contact conditions (Hashin and Shtrikman 1963) in a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

The idea that the pressure-acoustic velocity connection can be characterized by exponential function is well-known but the developed empirical models are based on mathematical curve fitting, however the physical meaning is unclear (Ji et al. 2007). To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper we present a quantitative petrophysical model, which explains the mechanism of pressure dependence of P and S wave velocity as well as describes well the acoustic hysteresis.

The pressure dependent velocity model in case of pressurization and depressurization cycles

The phenomenon is well-known that wave velocity is increasing with pressure directly and was explained on various rock mechanical studies. Following Birch's (1960) qualitative considerations we assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure -from the unloaded state-, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Therefore we introduce the parameter V as the unit pore volume of a rock. If a stress increase $d\sigma$ is created in a rock let us assume that the change of pore volume dV is directly proportional to the applied stress increase $d\sigma$ and also the pore volume V. One can describe the two assumptions with the following differential equation

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \tag{1}$$

where λ_V is new material quality dependent petrophysical parameter (Dobróka and Somogyi Molnár 2012) and V_0 is the pore volume at stress-free state ($\sigma = 0$). The negative sign represents that with increasing stress the pore volume decreases ($\lambda_V > 0$). We assume also a linear relationship between the infinitesimal change of the P wave propagation velocity $d\alpha$ - due to stress increase - and dV

$$d\alpha = -\kappa_P dV, \tag{2}$$

where κ_P is a positive proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining this assumption with Eqs. (1-2) and solve the differential equation one can obtain

$$d\alpha = \kappa_P \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \rightarrow \alpha = K - \kappa_P V_0 \exp(-\lambda_V \sigma), \tag{3}$$

where K is an integration constant. At stress-free state (σ=0) the propagation velocity α_0 can be measured which is computed from Eq. (3) as $\alpha_0 = K - \kappa_P V_0$. With this result and introducing the notation $\Delta\alpha_0 = \kappa_P V_0$ Eq. (3) can be rewritten in the following form

$$\alpha = \alpha_0 + \Delta\alpha_0 \left(1 - \exp(-\lambda_V \sigma)\right) . \tag{4}$$

Eq. (4) provides a theoretical connection between the propagation velocity and rock pressure in case of pressurization. Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded, hence decreasing velocity is observed. This effect is outside of our present investigations.

To characterize the depressurization cycle, $v=V_0-V$ as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed pores start to open again, so decreasing velocity can be measured. Therefore we assume dv (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease $d\sigma$

$$dv = \lambda'_V v \, d\sigma \;\rightarrow\; v = v_m \exp\left(-\lambda'_V \left(\sigma_m - \sigma\right)\right), \tag{5}$$

where λ'_V is another new material characteristic constant (which differs from the previously introduced parameter λ_V) and v_m is the closed pore volume at maximum pressure value σ_m. Combining Eq. (2) and Eq. (5) by using the formulas $dV=-dv$ and $\kappa_P v_m = \Delta\alpha_m$ one can find

$$\alpha = \alpha_m - \Delta\alpha_m \left(1 - \exp\left(-\lambda'_V \left(\sigma_m - \sigma\right)\right)\right). \tag{6}$$

Eq. (6) shows the propagation velocity – pressure function of depressurization cycle. In the two limiting cases (at pressure value $\sigma=\sigma_m$ and $\sigma=0$) Eq. (6) gives α_m and $\alpha_1 = \alpha_m - \alpha v_m \left(1 - \exp\left(-\lambda'_V \sigma_m\right)\right)$ respectively, (here the notation $\alpha(0) = \alpha_1$ was used). This gives the formula (similar to Eq. (4))

$$\alpha = \alpha_1 + \Delta\alpha_1 \left(1 - \exp(-\lambda'_V \sigma)\right) , \tag{7}$$

with the notation $\Delta\alpha_1 = -\alpha v_m \exp\left(-\lambda'_V \sigma_m\right)$.

P wave velocity measurements

The pressure dependent velocity model was tested on longitudinal wave velocity data sets. The pulse transmission technique was used for P wave velocity measurements. We performed measurements on many different sandstone samples which were subjected to uniaxial stresses by an automatic acoustic test system (Fig. 1a). It contains a load frame (max. 300KN) and a pressure generator (max. 80MPa). Wave velocities - as a function of pressure - were measured at adjoining pressures during pressurization and depressurization cycles. To avoid the destruction of the samples we loaded them only up to the 1/3 of uniaxial strength. One typical test result (Sample 1: fine-grained sandstone) is presented in the paper. Our measurements showed that the longitudinal velocity is increasing with pressure. Moreover a slight difference between the characteristics of the pressurization and depressurization curves was found that can be explained by the phenomenon of acoustic hysteresis. The explanation of acoustic hysteresis by Birch (1960) was followed: pores closed during pressurization do not reopen completely during depressurization; there is always a certain amount of irreversibility. This irreversibility in our model is denoted by two different parameters λ_V and λ'_V characterizing the pressurization and the depressurization cycles, respectively.

Extension of the model to S wave velocity

Since the base of the model is the change of pore volume (which is independent of the direction of loading) it can be applied also in case of S waves. Following the same procedure similar model equations can be obtained e.g. for the pressurization cycle the model contains parameters β_0, $\Delta\beta_0$, λ_V.
At present we are not able to measure S wave velocity therefore we tested our theory on velocity data published in literature (Gomez et al. 2010). P and S wave velocity data measured by the pulse

transmission technique on Oligocene Fountainebleau sandstone marked A11 (porosity was 7%) was processed.

Inversion results

Proving the validity and applicability of the introduced velocity model, we present the interpretation of measurement data of the described samples. The parameters appearing in the model equations (in case of pressurization and depressurization) valid also for P and S wave can be determined by processing measurement data based on joint inversion method (the Least Squares Method). The inversion results for each sample can be seen in Table 1. For the characterization of the accuracy of inversion estimates, the measure of fitting in data space (D(%)) and mean spread (S) was calculated (Table 1) at the end of the inversion procedure. It can be seen that the data misfits were small and the mean spread values indicate that the parameters are in moderate correlation, but the inversion results are reliable. These results confirm the accuracy of the inversion estimates and the feasibility of the developed petrophysical model. The application of the suggested model resulted in approximately the same data misfit on several sandstone samples.

Table 1 *Model parameters, RMS and mean spread of pressurization and depressurization cycles estimated by joint inversion using the developed model.*

P wave								
Sample	**Pressurization**			**Depressurization**			**D (%)**	**S**
	α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ_V (1/MPa)	α_1 (km/s)	$\Delta\alpha_1$ (km/s)	λ'_V (1/MPa)		
1	1,96	1,42	0,138	2,13	1,19	0,323	1,62	0,51
A11	3,26	1,93	0,084	3,63	1,54	0,148	5,51	0,45
S wave								
Sample	**Pressurization**			**Depressurization**			**D (%)**	**S**
	β_0 (km/s)	$\Delta\beta_0$ (km/s)	λ_V (1/MPa)	β_1 (km/s)	$\Delta\beta_1$ (km/s)	λ'_V (1/MPa)		
A11	2,17	1,25	0,091	2,48	0,95	0,126	6,64	0,46

With the estimated parameters the velocities can be calculated (separately at pressurization and depressurization) at any pressure by substituting them into Eq. (4) or Eq. (7). The results are shown in Fig. 1b and Fig. 2, where the solid line shows the calculated velocity-pressure function produced by the velocity model, while symbols represent the measured data. The figures show that the calculated curves are in good accordance with the measured data proving that the petrophysical model describing the acoustic hysteresis applies well in practice in case of also P and S waves.

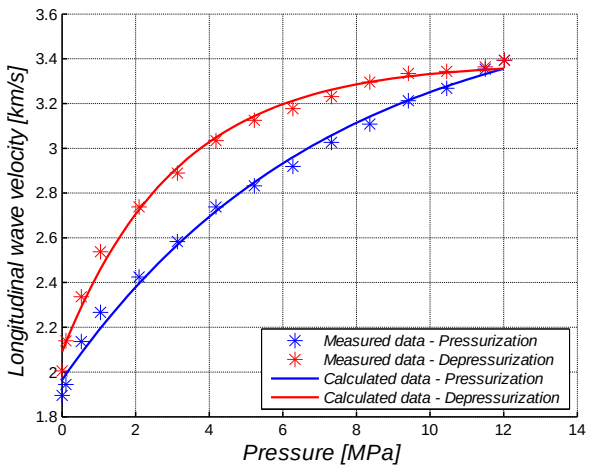

Figure 1 a.) *Automatic acoustic test system.* ***b.)*** *P wave velocity as a function of pressure of Sample 1 in case of pressurization and depressurization.*

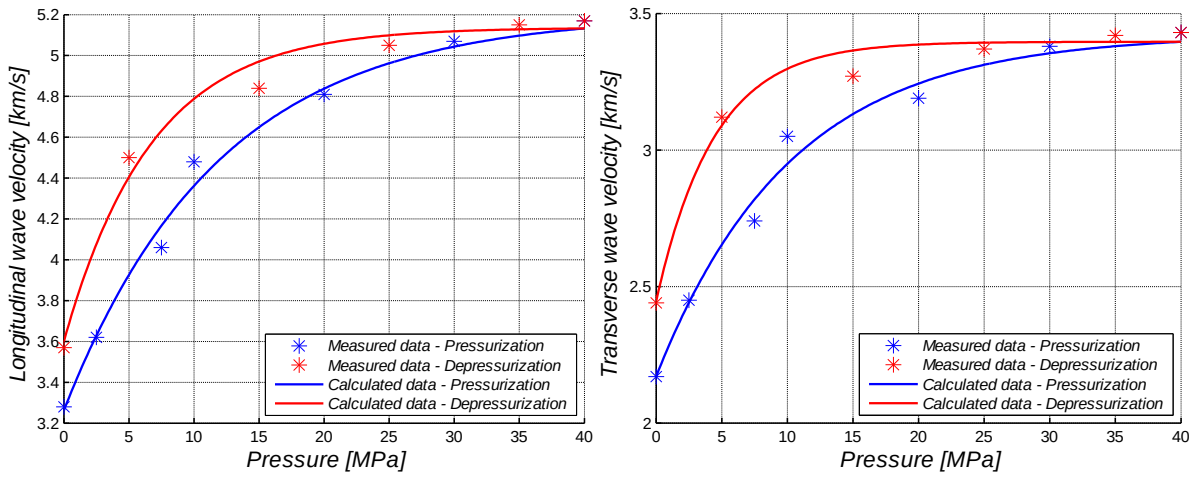

Figure 2 *P and S wave velocities as a function of pressure of Sample A11in case of pressurization and depressurization. Data obtained from Gomez et al. (2010).*

Conclusions

We presented a new petrophysical model describing the acoustic hysteresis which provides the connection between the propagation velocity of acoustic wave and rock pressure, both in case of pressurization and depressurization periods. The model (valid only in reversible/elastic range) is based on the idea that pore volume changes with pressure. Based on the model the acoustic hysteresis can be expressed by two different parameters because the closed pores do not reopen entirely during depressurization. The suggested model was applied to acoustic velocity data measured on core samples. Since the base of the model is the change of pore volume it could be applied also in case of S waves with success. By means of inversion-based data processing, the model parameters were determined from measurement data, thus, calculated data could be produced by the implementation of the petrophysical model in the forward problem. The calculated data match accurately with measured data proving that the petrophysical model describes well both pressurization and depressurization cycles. As it was shown the data misfit was small which supports the reliability of the inversion results and the accuracy, feasibility of the developed petrophysical model.

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