Low-Reynolds number flow around a cylinder following a figure-8-path – effect of direction of orbit

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Abstract
In this numerical study a circular cylinder is made to follow a figure-8 path. Time-mean and rms values of force coefficients and mechanical energy transfer are investigated versus frequency ratio at Reynolds numbers of 200, 250 and 300. Results differ substantially depending on the direction of orientation. Mechanical energy transfer was found to be always positive when the upper section of the figure-8 path moved in an anticlockwise direction, which may lead to vortex-induced vibration, and always negative for a clockwise path.

1. Introduction
Fluid-structure interaction, while a frequently studied topic, can be incredibly complex and researchers are always finding new physical phenomena in the wake of a bluff body. Flow around cylinders oscillating in transverse or in-line directions to the main stream is has long been in the focus of attention due to its practical importance.

Cylinder motion and the flow around a cylinder often cannot be modelled by one-degree-of-freedom (1-DoF) cylinder motion, although this is sufficient for pure transverse or in-line oscillation. In real life, typically components of both transverse and in-line motion occur (two-degree-of-freedom motion).

Studies dealing with 2-DoF forced cylinder motion basically fall into two groups: the first is when the frequencies are identical in x and y direction \((f_x=f_y)\), leading to an elliptical path (Didier and Borges, 2007; Baranyi, 2008). The second is when the frequency of in-line oscillation is double that of the transverse oscillation \((f_x=2f_y)\). In this case, quite a variety of cylinder paths can be produced by altering the phase difference between the two motions \(\Theta\). At each extreme \((-\pi/2, \pi/2)\) an arc is produced, although they curve in opposite directions, while in the middle \((\Theta=0)\) a symmetrical figure 8 is formed. As \(\Theta\) moves away from zero, the two lobes of the figure 8 become distorted. According to Sanchis et al. (2008), the figure-8 motion of a cylinder occurs when the mass of a freely oscillating body is close to that of the displaced fluid. Jeon and Gharib (2001) suggest that the phase value tends to vary within 45 degrees of zero for free vibration cases. Their experimental investigation of 2-DoF forced cylinder motion emphasised the importance of the phase difference between the two motions.

Numerical studies often place a cylinder in forced motion in order to gain an approximation of fluid-structure interaction. While this results in a simplified model, and a direct relationship between free and forced vibration is difficult to confirm (Williamson, 2004), it is a convenient approach to begin investigation of this complex phenomena.

Perdikaris et al. (2009) investigated flow around a mechanically oscillated cylinder following a figure-8 path at Reynolds number Re=400 while varying the transverse amplitude of oscillation. Their study looked at the power transfer parameter for two frequency ratios of 0.5 and 1. They found that the orientation of the motion (clockwise or anticlockwise orbit of the upper lobe) influences the results, generally leading to higher force coefficients and power transfer for the anticlockwise orientation.

Peppa et al. (2010) carried out a similar investigation for Re=400 at frequency ratios of 0.9, 1 and 1.1 at two different \(A_x/A_y\) ratio values. They also carried out computations for both directions of orbit and found that an anticlockwise orbit in the upper lobe resulted in a positive power coefficient, meaning an increased chance of vortex-induced vibration for a cylinder in free vibration.

An earlier numerical study of mine (Baranyi, 2010) dealt with forced figure-8 motion for Re=150, 200 and 250 against frequency ratio, for clockwise orientation only. One sudden change in vortex structure was identified for Re=250 only. The current numerical investigation examines the behaviour of a mechanically oscillated cylinder in a uniform flow, for both directions, at
Re=200, 250 and 300. Time-mean and rms values of force coefficients and mechanical energy transfer values for a cylinder following a symmetrical figure-8 path are compared by the direction of orbit in order to determine the influence of direction on flow behaviour.

2. Governing equations and computational setup
The dimensionless governing equations for an incompressible constant property Newtonian fluid flow around a moving circular cylinder are the two components of the Navier-Stokes equations, the continuity equation and pressure Poisson equation written in a non-inertial system fixed to the cylinder (not shown here due to lack of space; for further details see Baranyi (2008)).

Using boundary-fitted coordinates the governing equations and the boundary conditions are transformed into the computational plane and the equations are solved using the finite difference method. The 2D in-house code developed by the author was tested thoroughly against available experimental and computational results for stationary and moving cylinders (Baranyi, 2008).

Three Reynolds numbers of 200, 250 and 300 were investigated. During the whole investigation the oscillation frequencies are set at \( f_y = 2 f_x \), and oscillation amplitudes were kept constant at \( A_x = 0.14 \) and \( A_y = 0.5 \) to ensure a slender figure-8 path. Computations were repeated for different frequency ratios of \( f_y / S_{0x} \), where \( S_{0x} \) is the Strouhal number belonging to a stationary cylinder at the given Reynolds number. Only locked-in cases are investigated. For further details see Baranyi (2008). All computations are carried out for both directions of orientation. The inertial forces originated from the accelerating cylinder are removed, so only “fixed-body” coefficients (Baranyi, 2005) are shown in this paper.

Mechanical energy transfer between the fluid and the cylinder \( E \) is calculated from the following formula (see Baranyi, 2008):

\[
E = \frac{2}{\rho U^2 d^2} \int_0^T \mathbf{F} \cdot \mathbf{v}_d \, dt = \int_0^T \left( C_D \, v_{0x} + C_L \, v_{0y} \right) \, dt.
\]

3. Computational results and discussion
In Figure 1(a) it can be seen that the time-mean (TM) values of lift \( C_l \) for the anticlockwise direction of the upper lobe (filled signals) are zero for all three Reynolds numbers, just like for a transversely oscillating cylinder. The clockwise (cw) curves (empty signals) show one jump each for \( Re=250 \) and \( 300 \), showing a sudden switch in vortex structure. These curves are similar to those for in-line cylinder motion, and the vortex switch is probably due to a symmetry-breaking bifurcation (Crawford and Knobloch, 1991). The rms of lift, shown in Figure 1(b), also shows different behaviour depending on direction, with the clockwise curves changing gradually, and the anticlockwise (aclw) exhibiting a different pattern. The same tendencies were found for the TM and rms of torque coefficients (not shown here).

![Figure 8-shape motion; \( fy=f_x/2 \)](a)

![Figure 8-shape motion; \( fy=f_x/2 \)](b)

Figure 1: Time-mean (a) and rms (b) values of lift coefficient vs. frequency ratio for clockwise (\( Re=200, 250, 300 \)) and anticlockwise (\( Re=200a, 250a, 300a \)) orbit
Figure 2 shows the TM and rms of the drag coefficient $C_D$. Once again, the TM values are higher for the aclw case, although rms values are lower than those of the clw curves. Base pressure coefficients (not shown) show basically the same tendency for both TM and rms values.

![Figure 2](image)

(a) Figure 2: Time-mean (a) and rms (b) values of drag coefficient vs. frequency ratio for clockwise (Re=200, 250, 300) and anticlockwise (Re=200a, 250a, 300a) orbit

The mechanical energy transfer $E$ between the fluid and the cylinder within one motion period (based on $f_y$) is given in Fig. 3. For clw, it can be seen that $E$ curves are close to each other and always negative within the lock-in domain, while the aclw curves are always positive. These findings are in agreement with those of Perdikaris et al. (2009). The positive energy transfer means that cylinder motion is amplified by the fluid, which can lead to vortex-induced vibration (VIV). As can be seen in Fig. 3, $E$ is approximately constant over $f_y/\text{St}_0=0.8$ for all three Re values for the aclw case.

![Figure 3](image)

Figure 3: Mechanical energy transfer vs. frequency ratio for two directions

Some results of a pre- and post jump analysis are shown in Figs. 4 and 5 for the clockwise case at Re=300 (no jumps were found in any of the aclw curves). The pre- and post jump frequency ratio values are 0.8268 and 0.8270. Figure 4 presents ($C_D, C_L$) limit cycle curves for the two values. While perhaps not obvious at first sight, the two curves are mirror images, supporting the idea that the vortex switch is due to symmetry-breaking bifurcation. In Fig. 5 the vorticity contours pre- and post-jump for clw orientation are shown for the same (right-most) cylinder position. It can be seen that the vortex pattern has switched to a near-mirror image: before the jump double vortices are in the lower row but then switch to the upper row.

![Figure 4](image)

Figure 4: Limit cycles ($C_D, C_L$), Re=300, before jump (thick line) and after (thin line)
4. Conclusions

For a cylinder following a figure-8 path, the orientation is found to have a major effect on all of the parameters investigated. This is especially striking in the case of mechanical energy transfer, where an anticlockwise orbit leads to positive values, enhancing cylinder motion. Switches in vortex structure were found only for some clockwise cases.

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