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# MANUFACTURING PARAMETERS DETERMINATION ON BALL NUT GRINDING

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**Abstract:** This paper presents a method on the determination of tool profile for internal grinding of ball-nut. The solution is based on the so-called derivation theory, where the parameters of the tool (tilt angle, profile points, approximated profile curve) are determined by numerical methods. The final approximating tool profile is an ellipse-arc where the data points for the approximation are the intersection points of the surface intersection curves. Points of tool profile curves are determined by an initial value problem of ordinary differential equation system (ODE-IVP). The fitting method of approximating ellipses uses a numerically stable noniterative algorithm.

***Keywords:*** *ball-nut, tool profile, approximation, grinding*

## 1. Introduction

Ballscrew mechanisms are widely used in machine tools and the demand for high-lead ballscrews is increasing due to the high-speed manufacturing. The shape of the ball-nut profile is a gothic arc, which is a symmetrical combined curve of two arcs with equal radius and distance between their centres. These types of balls-nuts are generally manufactured by form grinding (Figure 1), where the grinding tool has corresponding profile [5], for high precision ballscrews lapping techniques are used as well [4]. In case of long and high lead threaded ball-nut the grinding wheel is not tilted at the lead angle of the thread to avoid the collision between the quill and workpiece.

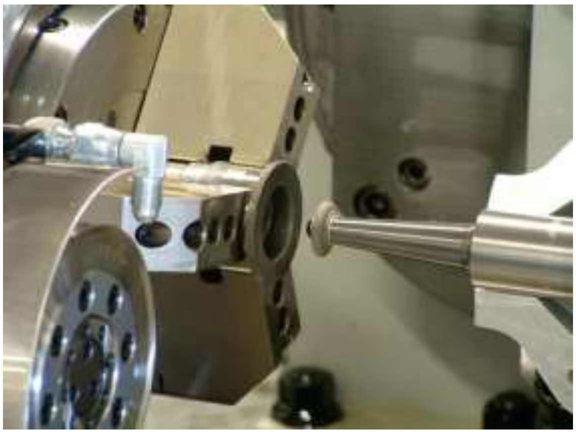

*Figure 1. Internal ball-nut grinding with conical tool holder*

Due to these conditions the profile obtained is not gothic-arc, because the grinding wheel tends to overcut the thread surface, this problem is well defined i.e. worm and gear drives [1–3]. There are known methods to obtain the tool surface geometry. Analytical solutions for profiling tools generated by surface enveloping are common and have been used for decades. These solutions are based on the fundamental theorems of the surfaces enveloping such as Olivier's second theorem and Gohman's fundamental theorem [3], [10]. Also, frequently used is Nicolaev's theorem [7], based on the helical movement decomposition. The minimum distance method [9] and the in-plane generating trajectories method [13] are also known profiling methods of this type of tools.

In case of long threaded ball-nut the setting of optimum tilt angle is not possible due to the collision of quill and workpiece (Figure 2). This limits the length of ball-nut which is manufactured. Generally, the length of a ball-nut is specified in advance. In the presented method the quill-inclination is reduced not to meet the internal surface of the workpiece within the specified ball-nut length. For this purpose, the grinding wheel has to be modified with a proper profile, which generates the gothic-arc thread to be obtained [5].

## 2. Tool tilt angle determination

The tool tilt angle determination is based on a collision detection, or minimum distance computation between the solid bodies of quill and workpiece. Collision detection between cylindrical bodies is widely used in three dimensional mechanical systems, for example machine tools, robots, different mechanisms. Detecting of collision between cylindrical rigid bodies were developed using line geometry by Ketchel and Larochelle [6]. Distance computation between cylinders has four different types according to their three dimensional positions in space (line-line, line-circle, circle-disk and circle-circle distance). Fast and accurate computation method was developed by Vranek [14]. To determine the maximum tilt angle for the grinding the minimum distance determination is required between the tilted quill axis and the edge of the ball-nut represented as a circle (Figure 2).

Determination of minimum distance between the quill and the ball-nut is equivalent with the computation of the distance between the quill axis and the circular edge of the ball-nut.

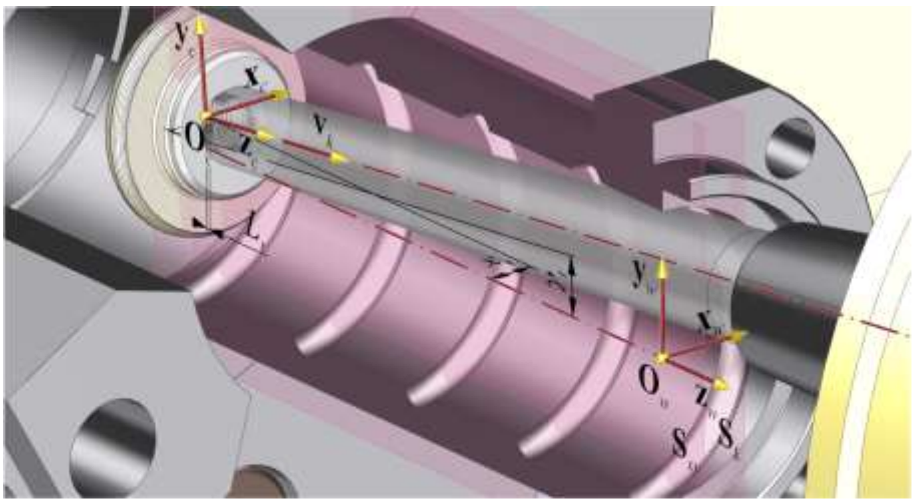

*Figure 2. Spatial position of tool-workpiece*

The equation of the workpiece circular edge can be written as

$$\mathbf{P}_A = \mathbf{C} + \frac{D_{szt}}{2}\left(\cos(\phi)\mathbf{u} + \sin(\phi)\mathbf{v}\right) \tag{1}$$

where $\phi \in [0, 2\pi]$, $\mathbf{P}_A$ is point of the circle, $\mathbf{C}$ is centre of the circle, $D_{szt}$ is diameter of the quill and $\mathbf{u}$ and $\mathbf{v}$ are unit vectors in the plane containing the circle. The distance between the quill axis and the circular edge is

$$d_{\min}(\gamma_t, t) = b_h - \left(\frac{D_{szt}}{2}\right)^2 + \left|\mathbf{C} - \mathbf{P}_M\right|^2 - D_{szt}\frac{\mathbf{Q} - \mathbf{C}}{\left|\mathbf{Q} - \mathbf{C}\right|} \cdot (\mathbf{C} - \mathbf{P}_M) \tag{2}$$

where $\mathbf{P}_M$ is a point on the quill axis and $\mathbf{Q}$ is the projection of $\mathbf{P}_M$ on the circle plane and $b_h$ is a safety gap between the quill and the ball-nut. Applying the expressions from [12] a nonlinear equation system can be formulated for the unknown parameters. Differentiate equation (2) with respect the quill axis parameter

$$\frac{\partial d_{\min}(\gamma_t, t)}{\partial t} = 0 \tag{3}$$

the minimum distance between the quill and workpiece can be obtained. Equation (2) and (3) form a quartic nonlinear equation system. Its solutions can be found by root finder algorithms (generally *Newton–Raphson* method is used). In case of conical quill the distance equation is reformulated to

$$d(\gamma_t, \phi) = \left|\mathbf{P}_A - \mathbf{v}_a\right| \sin\left(\cos^{-1}\left(\frac{(\mathbf{P}_A - \mathbf{v}_a)\mathbf{v}_k}{\left|\mathbf{P}_A - \mathbf{v}_a\right|\left|\mathbf{v}_k\right|}\right) - \beta\right), \tag{4}$$

where $\mathbf{v}_a$ the direction vector of the conical surface generatrix, $\beta$ half opening angle of the cone. The minimum distance between the conical surface and the workpiece edge is

$$\frac{\partial d_{\min}(\gamma_t, \phi)}{\partial \phi} = 0. \tag{5}$$

Due to the reduced quill-inclination the grinding wheel has to be modified with the proper profile depends on the workpiece parameters. In the next section the method of tool profile generation is described.

## 3. Tool profile generation

In this section a numerical method for the determination of tool profile of grinding tool is described. The surface intersection method is based on the solution of system of ODEs. The parametric equation of the internal gothic arc ball-nut surface is

$$\mathbf{S}_b(u, v) = \mathbf{h}(u) + R_{pr}\left[\mathbf{b} \cdot \sin(v) - \mathbf{n} \cdot \cos(v)\right], \tag{6}$$

where $\mathbf{h}(u)$ is the parametric equation of helical curve of swept surface, $\mathbf{b}$ is the binormal, $\mathbf{n}$ is the normal vector of helical curve and $R_{pr}$ is the radius of gothic arc. The parametric equation of the tool plane is given by

$$\mathbf{S}_k(q, t) = \mathbf{O}_c + q \cdot \mathbf{z}_c + t \cdot \mathbf{x}_c. \tag{7}$$

The intersection curve of two surfaces is determined by

$$\mathbf{C}(u, v, q, t) = \mathbf{S}_b(u, v) - \mathbf{S}_k(q, t) = \mathbf{0}. \tag{8}$$

If a direction vector can be found such that is orthogonal to all gradients, than the intersection curve can be traced by following this direction. This orthogonal vector is determined by a modified *Jacobian* determinant

$$\mathbf{P}(u,v,q,t)=\det\begin{bmatrix}\mathbf{e}_u & \mathbf{e}_v & \mathbf{e}_q & \mathbf{e}_t \\ & J_C(u,v,q,t) & & \end{bmatrix}, \tag{9}$$

where $J_C(u,v,q,t)$ is the *Jacobian* of the intersection curve formed by equation (8). The above formula determines an *ODE-IVP* system. The initial value vector is determined by fixing one of the parameters and solving for the rest by *Newton–Raphson* method. After the appropriate starting point found the *ODE* system is solved numerically by 4–5 order *Runge–Kutta* algorithm [8].

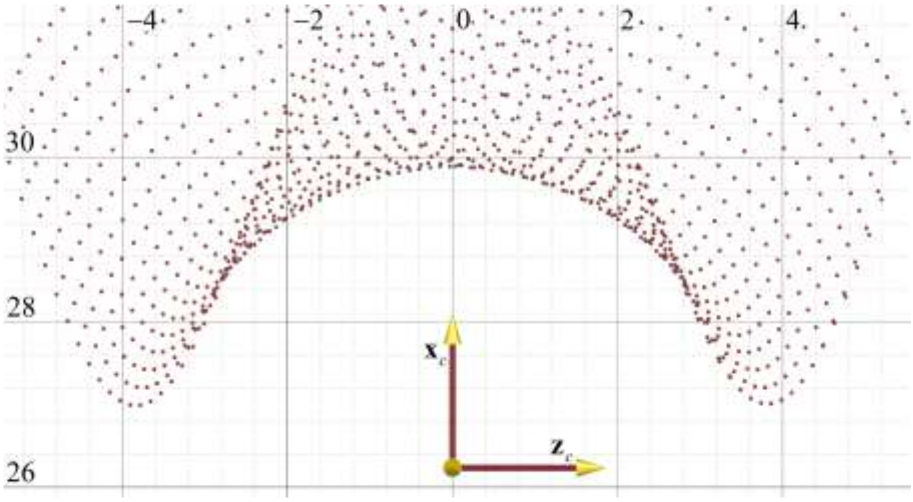


*Figure 3. Surface intersection points of Runge–Kutta algorithm*

The number of surface intersections determines the number of ODEs to solve (Figure 3). Obtaining the points of intersection curve an approximated ellipse-arc is determined by a numerically stable non-iterative algorithm, described in the next section.

Starting and ending points and the intersection points of the ellipse-arcs are requested to generate tool profile. Figure 3 shows the intersection points, the profile and noise points transformed into *xz plane* of the tool. Ellipses are special cases of general conics which can be described by an implicit second order polynomial

$$F(x,z)=ax^2+bxz+cy^2+dx+ez+f=0, \tag{10}$$

where *a, b, c, d ,e ,f*are coefficients of the conic and $b^2–4ac<0$ is a further constraint for ellipse. The *algebraic distance* $F(x,z)$ rewritten in vector form

$$F(\mathbf{x})=\mathbf{x}\cdot\mathbf{a}=0. \tag{11}$$

The fitting of a general conic to a set of points ($x_i$; $z_i$); $i = 1\ldots n$ may be approached by minimizing the sum of squared algebraic distances of the points to the conic which is represented by coefficients **a**:

$$\min\sum_{i=1}^{n}F(x_i,z_i)^2=\min\sum_{i=1}^{n}\left(F(\mathbf{x_i})\right)^2=\min\sum_{i=1}^{n}\left(F(\mathbf{x_i}\cdot\mathbf{a})\right)^2 \tag{12}$$

Equation (12) can be solved directly by the standard least squares algorithm, but the result is a general conic. In specific ellipse case further constraint required to fit an ellipse to the filtered profile points [15]. An improved fitting algorithm is proposed and implemented in the appropriate MATLAB code [11].

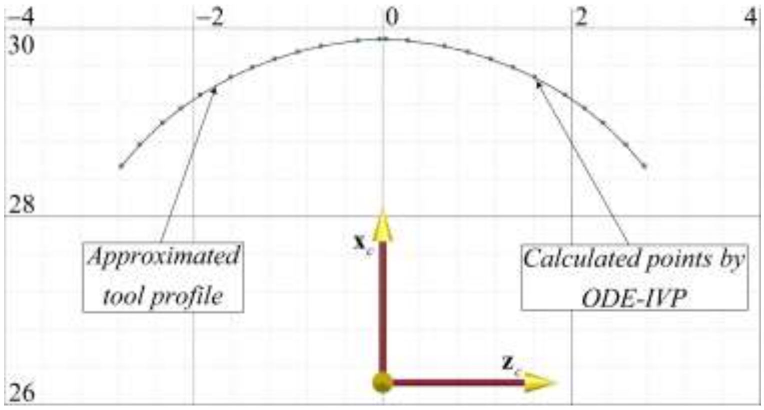


*Figure 4. Points for profile approximation,* d = 50mm, p = 30mm

There are two noise sections among these collected points which have to be filtered out. The limiting point of undercutting is calculated by

$$\left|\mathbf{B}(t)_{i-1,i} - \mathbf{F}_1\right| + \left|\mathbf{B}(t)_{i-1,i} - \mathbf{F}_2\right| - 2a = 0 \quad (13)$$

where $\mathbf{B}(t)_{i-1,i}$ is the $i$-th cubic *Bezier* curve ($t = 0...1$, $i = 1...n$ , $n$ the number of intersection curves), $\mathbf{F}_1$ and $\mathbf{F}_2$ focus points of fitted ellipse on the intersection points of ellipse-arcs between the minimum point of intersection points and ending point of tool profile (this point calculated from ball-nut parameters), $a$ semi-major axis of the fitted ellipse-arc. The solution of nonlinear equation (13) is determined by *Newton–Raphson* method. The final approximated tool profile is generated by mirroring of the two approximating cubic *Bezier* curve and ellipse-arc (Figure 4).

## 4. Summary

This paper presents numerical applications for the profiling of grinding tools for the generation of gothic-arc helical surfaces with constant pitch, based on derivation theory and surface-surface intersection. The proposed methods used the capabilities of the MATLAB mathematical computing environment. The results was obtained in numerical form and confirmed by graphically. The final approximating tool profile is an ellipse-arc where the data points for the approximation was the intersection points of the surface intersection curves determined by an initial value problem of ordinary differential equation system (ODE-IVP).

The accuracy of the results depend on the Runge–Kutta step size used for to solve the ODE-IVP. The method is generic that it can be used for different geometrical shapes and different tool profile.

## 5. Acknowledgement

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