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CONNECTION BETWEEN THE LEAKING AND THE VISCOELASTIC BEHAVIOR OF FLANGE GASKETS

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Abstract. This paper presents a measurement and calculation method to determine the stress relaxation function parameters of a flange gasket which has viscoelastic behavior. This is important because it has a strong connection to the leakage of vessels.

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Keywords: gasket, stress relaxation, generalized Maxwell model, penalty function.

1. Introduction

Operation of closed systems often causes isolation problems. In this case air contaminants may leak into the working area or into the environment. Flange-gasket looseness is the source of the leaking most times. This paper points out the main cause of the leakage of a soft PTFE (polytetrafluoroethylene) covered textile gasket between flange joints. A measurement apparatus has been created to examine PTFE covered gaskets. With the help of this apparatus the stress and deformation in the gasket can be measured.

2. Gasket investigation unit

The measurement unit has been created for gasket measuring is shown in Figure 1. The main parts of the investigation unit are:

1. tension tester (load capacity: 25 kN),
2. load cell,
3. flange,
4. gasket,
5. displacement transmitter,
6. A/D converter,
7. computer.

During the measurements the flange gasket is compressed by the tension tester. The compression stress and the gasket deformation (compressive strain) are recorded by the A/D logger-converter. When the stress reaches the maximum, the increment of the stress is stopped. With this procedure we can simulate flange-joint gasket deformation and stress relaxation.

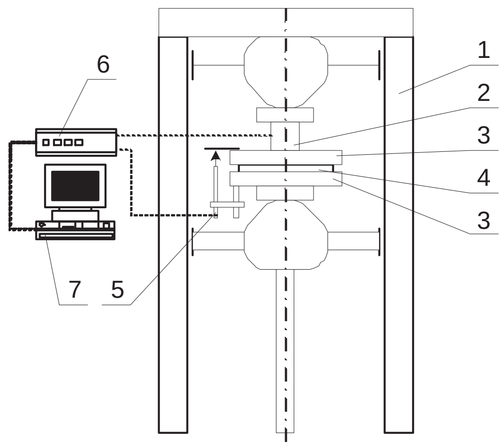

Figure 1. Measurement apparatus

If the gasket is not working properly leakage can occur. This happens if the gasket parameters are not correct or the gasket is damaged. If the gasket stress cannot reach the required value or the stress is reduced below the required value a leaking process can start. Due to the leakage, the air contaminating mass flow spilling into the atmosphere is determinable [1].

3. Mechanical model of flange connection

The simplified mechanical model of flange connection is shown in Figure 2. The base load of the flange is the bending momentum. This load arises from the bolt force, the inner pressure force and the gasket force. The flange and the gasket forces are different in operation state and assembling state. The inner pressure forces are zero in the assembling state. In the present case the gasket force is higher than the other. The minimum bolt force in assembling state can be calculated by:

$$W_A = \pi b \, G \, y, \tag{1}$$

where b is the effective gasket width, G is the diameter of the gasket center line, y is the minimal gasket stress.

If the applied bolt force is lower than W_A (calculated with (1)) the gasket is not working acceptably and it will leak.

In the operational state the bolt force has to be higher than in assembling state. This force can be calculated with this equation:

$$W_{OP} = \frac{\pi}{4} G^2 \, P + 2\pi G m P, \tag{2}$$

where P is the pressure, m is the gasket parameter. This gasket parameter depends on the material of the gasket. Table 1 shows the gasket parameter and the minimal gasket stress for different types of gaskets:

Table 1. Typical gasket parameters and minimal stresses

Type of gasket	Gasket parameter, m	Gasket minimal stress, y, MPa
Rubber	0.5 - 1	0 - 1.4
PVC	1.5	1.2
PTFE	2 - 2.75	1.2 - 1.6
Rubber with textile	1.25	2.75
IT sheet	2.25 - 2.75	15 - 25
Waxed seal	2.5 - 3.5	25 - 52

The effective gasket stress depends on the bolt force, the gasket parameter, the gasket minimal stress and of course the geometry of the flanged connection. If

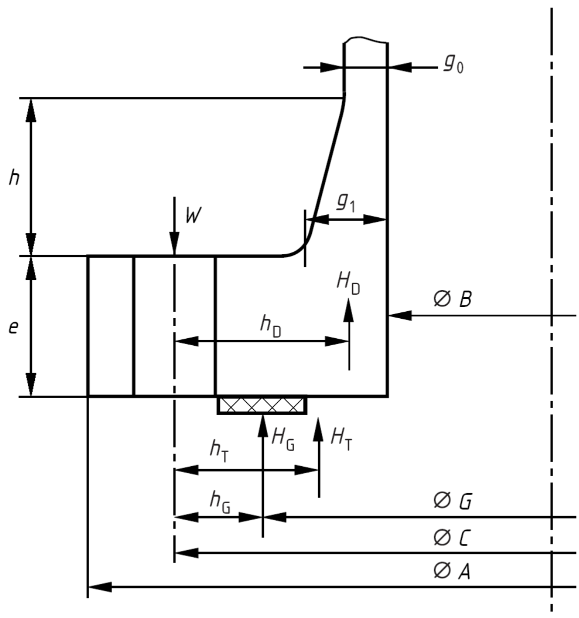

Figure 2. General mechanical model of flange connection

the gasket material shows viscoelastic or viscoplastic properties, the gasket stress also depends on the time.

4. Generalized Maxwell Model

The material of the PTFE covered textile gasket shows viscoelastic properties. The viscoelastic material model is described by rheological elements. The generalized Maxwell model [2, 5], shown in Figure 3, is used for describing the material behavior of the gasket.

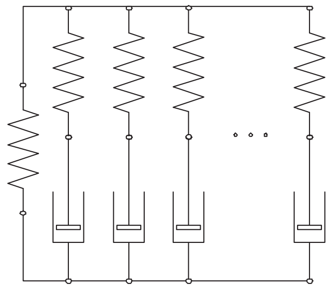

Figure 3. Generalized Maxwell model

Assuming that gasket deformation is only in axial direction, there is no radial deformation. Consequently, only volumetric stresses occur in the gasket. This linear viscoelastic behavior is commonly using the Boltzmann superposition integral [3]

$$\sigma(\tau) = \int_0^\tau K(\tau - \tau') \frac{\partial \epsilon}{\partial \tau'} \partial \tau', \tag{3}$$

where K is the relaxation function, τ is the time, ϵ is the deformation. The relaxation function is approximated with the following formula:

$$K(\tau) = K_\infty + K_0 \sum_{k=1}^{m} w_k e^{-\frac{\tau}{\tau_k}}. \tag{4}$$

The $\sigma(\tau)$ stress-function is approximated with

$$f_k(t) = A + B \sum_{j=1}^{m} w_j e^{-t/\tau_k}, \tag{5}$$

where A is the residual stress, B is the relaxation factor, w_j is the weighting coefficient, m is the number of the Maxwell elements, τ_k is the relaxation time of one of the Maxwell elements.

According to the investigation results, in case of $m = 3$, the approximation is suitable. The least squares method is used in the approximation process:

$$F = \sum_{i=1}^{n} (f_{ki} - f_{mi})^2 \to \min, \tag{6}$$

where n is the number of the measuring points, f_{ki} the approximated stress-function, f_{mi} is the measured stress values.

Derivative of function (6) with respect to the variable A:

$$\frac{\partial F}{\partial A} = 2\sum_{i=1}^{n}(f_{mi} - f_{ki}). \tag{7}$$

Derivative of function (6) with respect to the variable B:

$$\frac{\partial F}{\partial B} = 2\sum_{i=1}^{n}(f_{mi} - f_{ki}).\sum_{j=1}^{m}(w_j e^{-\frac{t_i}{\tau_j}}). \tag{8}$$

Derivative of function (6) with respect to the variable w_k, where k=1,2,3:

$$\frac{\partial F}{\partial w_k} = 2\sum_{i=1}^{n}(f_{mi} - f_{ki})\left[Be^{-t_i/\tau_k}\right]. \tag{9}$$

Derivative of function (6) with respect to the variable τ_k, where k=1,2,3:

$$\frac{\partial F}{\partial \tau_k} = 2\sum_{i=1}^{n}(f_{mi} - f_{ki})\left[Bw_k\frac{t_i}{\tau_k^2}e^{-t_i/\tau_k}\right]. \tag{10}$$

The eight nonlinear equations involve eight unknown parameters. The nonlinear equation system in a reduced form is the following:

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right] = 0. \tag{11}$$

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right]\left[w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right] = 0. \tag{12}$$

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right]\left[Be^{-\frac{t_i}{\tau_1}}\right] = 0. \tag{13}$$

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right]\left[Be^{-\frac{t_i}{\tau_2}}\right] = 0. \tag{14}$$

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right]\left[Be^{-\frac{t_i}{\tau_3}}\right] = 0. \tag{15}$$

$$\sum_{i=1}^{n}\left[A + B\left(w_1 e^{-\frac{t_i}{\tau_1}} + w_2 e^{-\frac{t_i}{\tau_2}} + w_3 e^{-\frac{t_i}{\tau_3}}\right) - f_{mi}\right]\left[Bw_1\frac{t_i}{\tau_1^2}\right] = 0. \tag{16}$$

$$\sum_{i=1}^{n}\left[A+B\left(w_1 e^{-\frac{t_i}{\tau_1}}+w_2 e^{-\frac{t_i}{\tau_2}}+w_3 e^{-\frac{t_i}{\tau_3}}\right)-f_{mi}\right]\left[Bw_2\frac{t_i}{\tau_2^2}\right]=0. \tag{17}$$

$$\sum_{i=1}^{n}\left[A+B\left(w_1 e^{-\frac{t_i}{\tau_1}}+w_2 e^{-\frac{t_i}{\tau_2}}+w_3 e^{-\frac{t_i}{\tau_3}}\right)-f_{mi}\right]\left[Bw_3\frac{t_i}{\tau_3^2}\right]=0. \tag{18}$$

If this equation system is solved, we get the parameters of the approximation functions. During this minimization, the following equations should be satisfied:

$$\sum_{j=1}^{k} w_k - 1 = 0 \rightarrow h(X) = 0, \tag{19}$$

$$\begin{bmatrix} -A \\ -B \\ -w_1 \\ -w_2 \\ -w_3 \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \leq 0 \rightarrow g(X) \leq 0. \tag{20}$$

The following constrained-extremum problem should be solved in order to simplify:

$$\begin{gathered} F(X) \rightarrow \min, \\ h(X) = 0, \\ g(X) \leq 0. \end{gathered} \tag{21}$$

The relevant mathematical literature offers a lot of methods to solve (21). A penalty-function technique [4] is used to solve the problem. The following penalty function is used in the procedure:

$$\Theta(X,\sigma) = F(X) + \sigma\sum_{q=1}^{r} h_q^2(X) + \sigma\sum_{y=1}^{c}(max(g_y(X),0))^2. \tag{22}$$

The constrained-extremum problem (21) can be converted to an unconditional extremum problem with the help of the penalty function. The *Nelder-Mead* procedure, which is implemented in MATLAB, is used to solve the problem. For the σ sequence: $\sigma_k = 10^{k-1}$.

Figure 4 shows one of the approximated results.

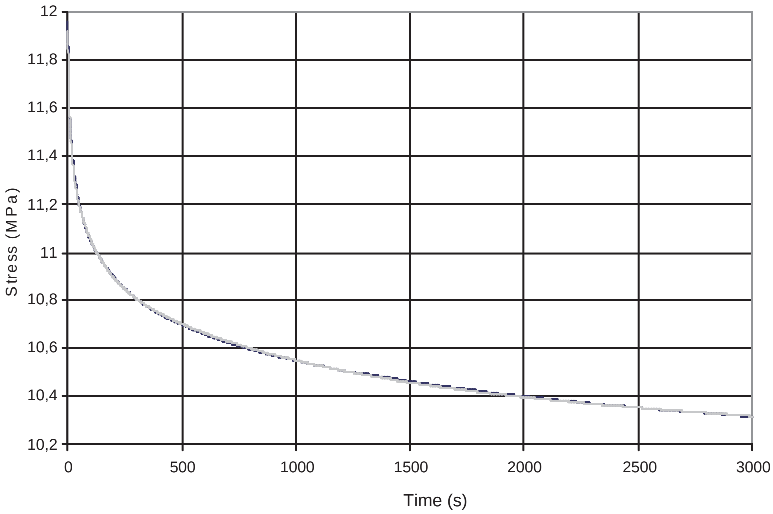

Figure 4. The measured and calculated stress

Measurements are made for different maximal gasket stress states. A summary of the approximation results are shown in Tables 2-4.

Table 2. Results for 3 MPa gasket loading

No.	A	B	w_1	w_1	w_1	τ_1	τ_1	τ_1	A/σ_{max}
1	2.03	0.58	0.37	0.28	0.35	19.9	556	7454	0.77
2	1.94	0.49	0.35	0.27	0.38	40.1	634	7388	0.77
3	1.96	0.57	0.3	0.33	0.37	37	408	3765	0.74

Table 3. Results for 6 MPa gasket loading:

No.	A	B	w_1	w_1	w_1	τ_1	τ_1	τ_1	A/σ_{max}
1	4.96	1.62	0.35	0.27	0.38	72.4	918	11359	0.72
2	5.17	1.64	0.35	0.29	0.36	41.7	740	9902	0.72

Table 3. Results for 13 MPa gasket loading:

No.	A	B	w_1	w_1	w_1	τ_1	τ_1	τ_1	A/σ_{max}
1	11.36	2.34	0.41	0.26	0.33	43.9	717.5	8571	0.79
2	11.62	2.49	0.42	0.25	0.32	45.3	907	11042	0.79
3	10.7	2.31	0.41	0.27	0.33	64.3	930.3	9836	0.79
4	11.12	2.47	0.38	0.26	0.36	47.5	736.5	9137.8	0.78

In Tables 2-4 the last columns show by what percent the maximal gasket stress decreased after the relaxation process. In the worst case (common in engineering) the residual stress is 70 % of the maximal gasket stress. If this value does not reach the minimal stress of the gasket, leaking may happen.

5. Conclusion

The presented calculation and measuring method is suitable to describe the viscoelastic type gasket time-stress function and determine the residual gasket stress on account of the stress relaxation process. In the future the effects of the re-loading on the relaxation properties will be investigated.

References

1. Szepesi, G. and Ortutay, M.: *Mathematical model of gas leaking.* GÉP, **10-11**, (2004), 126-131. (in Hungarian)
2. Papoulia, K. D., Panoskaltsis, V. P. and I. Korovajchuk, I.: A class of models of 3D finite viscoelasticity *School of Civil and Environmental Engineering, Cornell University, Ithaca, USA.*
3. Del Nobile, M. A., Chillo, S,, Mentana, A. and Baiano, A.: Use of the generalized Maxwell model for describing the stress relaxation behavior of solid-like foods. *Journal of Food Engineering*, **78**, (2007), 978-983.
4. Galántai, A. and Jeney, A.: Numerical Methods. *Miskolc, University Press*, Miskolc, 1998. (in Hungarian)
5. González, J. A. and Abascal, A.: Linear viscoelastic boundary element formulation for steady state moving loads *Engineering Analysis with Boundary Elements*, **28**(7) (2004), 815-823.