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Effect of Pressure on Compression, Shear and Young's Moduli

A. Kiss (University of Miskolc), J. Somogyi Molnar* (University of Miskolc, MTA-ME Geoengineering Research Group), M. Dobroka (University of Miskolc, MTA-ME Geoengineering Research Group), T. Ormos (University of Miskolc) & Á. Gyulai (University of Miskolc)

SUMMARY

The rocks response as perfectly elastic materials in case of rapidly changing stresses. With the assumption of the Hooke body, the elastic moduli describe how rocks resist different deformations. Present investigations covered the examination of pressure dependence of compressional, shear and Young's moduli. As they can be calculated from the acoustic wave velocities (longitudinal and transverse) it is important to know accurately the velocity-stress function. Therefore the authors developed a petrophysical model, which gives the physical connection between the acoustic velocities and stresses. After estimating the model parameters by joint inversion, where the λ_V rock physical parameter is the common parameter, the velocities can be calculated at any arbitrary stresses and the pressure dependent elastic moduli can be derived. To prove the applicability of this method, we measured P and S wave velocities on sandstone samples with an automatic acoustic test system under uniaxial load. This paper includes one sample from these measurements together with literature data of a Berea sandstone sample. The results show that the misfits between measured and calculated data are small, the model can be applied well in practice.

Introduction

Elastic moduli such as compressional, shear or Young's moduli are important and often determined quantities. They describe how well rocks resist different deformations. The higher the moduli are the stiffer the material is. In case of rapidly changing stresses, such as those generated during propagation of acoustic waves, the rocks response as perfectly elastic materials. It means that they suffer strain during loading, but after unloading they perfectly recover their shapes. This is the assumption of the Hooke body, where the stresses are proportional to the displacements in case of small deformations. In isotropic, linearly elastic medium two constants are enough to describe the stress-deformation relationship. In the general form of the Hooke body these constants are the Lamé coefficients, but further elastic moduli can be introduced as well. Present study includes the investigation of compression, shear and Young's moduli and the pressure dependence that of parameters.

Effect of pressure on elastic moduli

In the frame of the perfectly elastic medium model Hooke's law can be written as

$$\sigma = E\varepsilon \, , \tag{1}$$

where ε is the strain of the rock caused by the applied stress σ and E is the proportionality factor, the elastic modulus. The strain produced by the acoustic waves can be considered elastic (Barton 2007). Depending on the direction of applying and measuring stress and strain, different elastic moduli can be defined. This paper includes the investigation of Young's and shear moduli beside the compression (bulk-K) modulus. Latter is defined as the ratio of the hydraulic stress to the volumetric strain. The shear modulus (G) is the ratio of the shear stress to the shear strain. Young's modulus (E) is considered the ratio of extensional stress to extensional strain. They can be calculated by the formulas in Eq. (2)

$$K = \rho\left(\alpha^2 - \frac{4}{3}\beta^2\right), \qquad G = \beta^2\rho \, , \qquad E = \beta^2\rho\frac{3\alpha^2 - 4\beta^2}{\alpha^2 - \beta^2} \, . \tag{2}$$

It can be seen from Eq. (2), that the elastic moduli can be derived if the acoustic wave velocities (longitudinal α and shear β) and the density of the medium (ρ) are available. Latter can be determined in laboratory and its pressure dependence is negligible in contrast to the effect of pressure on the velocities. Therefore one requires accurate velocity measurements to obtain pressure dependent elastic moduli.

Rock physical model for the pressure dependence of acoustic velocities

The authors developed a rock physical model which explains the physical relationship between the applied stress and the acoustic P and S wave velocities. The model is based on the idea formulated by Birch (1960). He assumed that the main reason for the increasing velocity under loading is the closure of pores. Hence the model law of our velocity model can be formulated by Eq. (3)

$$dV = -\lambda_V V d\sigma \, , \tag{3}$$

where dV is the change of unit pore volume, $d\sigma$ is the applied stress increase and λ_V is the proportionality factor, a new rock physical parameter. The negative sign represents that the increasing stress causes decrease in the pore volume.

We assume also linear relationship between the infinitesimal change of the appropriate propagation wave velocity dv (substitutable with the longitudinal or shear wave) and dV

$$dv = -\kappa dV \, , \tag{4}$$

where κ is a proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume.

Combining Eqs. (3-4) and solving the differential equations one can obtain

$$v = K - \kappa V_0 \, exp(-\lambda_V \sigma), \quad (5)$$

where K is an integration constant. At stress-free state ($\sigma = 0$) the propagation velocity v_0 and the pore volume V_0 can be measured and K can be computed from Eq. (5) as $v_0 = K - \kappa V_0$. With this result and introducing the notation $\Delta v_0 = \kappa V_0$ Eq. (5) can be rewritten in the following form

$$v = v_0 + \Delta v_0 \, (1 - exp(-\lambda_V \sigma)). \quad (6)$$

The introduced quantity Δv_0 means the difference between the velocities measured at maximum and zero stresses i.e. $\Delta v_0 = v_{max} - v_0$, in other words it is the velocity-drop caused by the presence of pores at stress-free state (Ji et al. 2007). The physical meaning of the parameter λ_V must be drawn up as well. The velocity-drop can be calculated at any arbitrary stresses as $\Delta v = v_{max} - v$. Substituting the formulas of Δv_0 and Δv, Eq. (6) can be written in the form of

$$\Delta v = \Delta v_0 \, exp(-\lambda_V \sigma). \quad (7)$$

At the σ^* characteristic stress $-\lambda_V \sigma^* = 1$ and therefore $\Delta v_0 = \Delta v / e$ i.e. the velocity-drop falls to $1/e$ of the "initial" velocity-drop. In addition the meaning of λ_V can be expressed as the logarithmic stress sensitivity of the velocity-drop $\Delta v = v_{max} - v$ (Dobróka and Somogyi Molnár 2012) as

$$S(\sigma) = -\frac{1}{\Delta v}\frac{d\Delta v}{d\sigma} = -\frac{d \, ln(\Delta v)}{d\sigma} \rightarrow \lambda_V = -\frac{d \, ln(\Delta v)}{d\sigma} = S \,. \quad (8)$$

By substituting the appropriate velocities the model equations describing the pressure dependence of longitudinal and shear waves can be obtained in the forms of Eq. (9)

$$\alpha = \alpha_0 + \Delta\alpha_0 \, (1 - exp(-\lambda_V \sigma)), \; \beta = \beta_0 + \Delta\beta_0 \, (1 - exp(-\lambda_V \sigma)). \quad (9)$$

If both P and S wave velocity data are available they can be processed in joint inversion procedure, since λ_V is a common parameter.

Case studies

To prove the applicability of our model and to derive the pressure dependent compressional, shear and Young's moduli, we performed measurements on many different sandstone samples by an automatic acoustic test system under uniaxial stresses. The test system includes a load frame, a pressure cell and a 2-channel ultrasonic device. The experimental setup is shown in Figure 1. The accuracy of the load frame (max. 300 kN) according to ISO 7500-1 is grade 1 up to 30 kN and grade 0.5 above. The linearity of the load cell is 0.05%. The samples were loaded linearly with 0.1 kN/s by the help of DION software. To avoid the failure of the samples we loaded them only up to 1/3 of uniaxial strength, for sample *A* it was 20 kN. The pulse transmission technique was used to determine the acoustic velocities. Transducers in the cell (1 MHz eigenfrequency, 35-mm diameter) were used to generate P and S waves. We applied 256-fold stacking to increase the signal/noise ratio. In this paper the results of sample *A* (own measurement) and a Berea sandstone sample chosen from literature (Winkler and Murphy 1995) are presented. Both cylindrical samples are sandstones, with density of 2620 and 2610 g/cm^3, respectively.

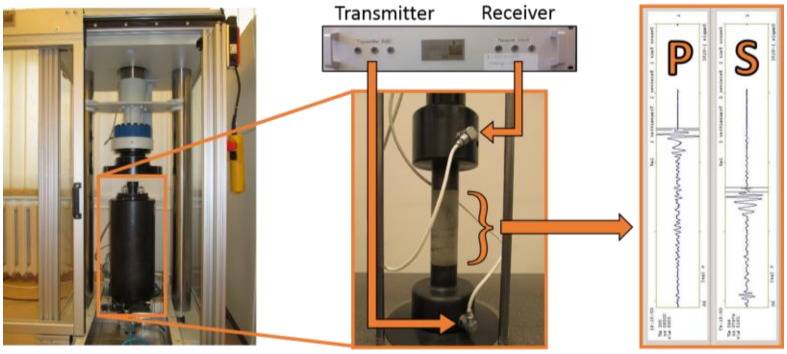

Figure 1 *Experimental setup. Left: load frame and pressure cell. Middle: ultrasonic device, sandstone sample between transmitter and receiver built in the pressure stamps. Right: P and S wave arrivals.*

Results

Since λ_V is a common parameter, the 5 model parameters ($\alpha_0, \Delta\alpha_0, \lambda_V, \beta_0, \Delta\beta_0$) of the two response equations formulated in Eq. (9) were determined by joint inversion process. The problem was overdetermined, therefore the Gaussian Least Squares Method was used. After determining the model parameters the velocities were calculated by Eq. (9) for the stresses in the region of interest. Based on these velocities the pressure dependent elastic moduli (K, G, E) were derived. The results are summarized in Figure 2 for sample *A* and in Figure 3 for the Berea sandstone. In both figures the upper graphs present the velocities, the lower ones show the elastic moduli. Dots mean the measured values (in case of elastic moduli they are calculated from the measured velocities), the lines represent the values calculated by inversion (in case of elastic moduli they are calculated from the velocities determined by inversion).

For the characterization of the inversion estimates, their errors are given as well. The variances of the model parameters are the elements of the main diagonal of covariance matrix in parameter space. The RMS values are calculated for the characterization of the accuracy of inversion estimates, the mean spread shows the reliability of the suggested petrophysical model. The variances, RMS and mean spread were calculated by the formulas given by Menke (1984) and their values are summarized in Table 1. It can be seen that the data misfits (RMS) were small (0.11-4.01%) and the parameters are in moderate correlation indicated by the mean spread.

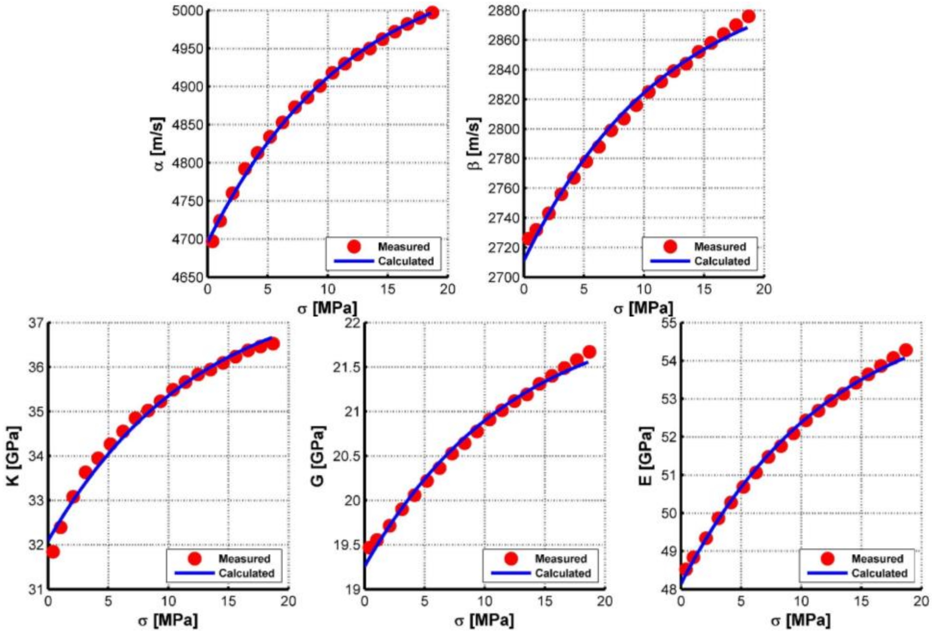

Figure 2 *Pressure dependent velocities and elastic moduli for sample A*

Table 1 *Estimated model parameters by joint inversion method, their RMS and mean spread values*

Sample	P wave		Common	S wave		D				S
	α_0 [m/s]	$\Delta\alpha_0$ [m/s]	λ_V [1/MPa]	β_0 [m/s]	$\Delta\beta_0$ [m/s]	v [%]	K [%]	G [%]	E [%]	[-]
A	4695.6 (±2.9)	379.6 (±7.8)	0.0844 (±0.0040)	2711.1 (±2.3)	198.6 (±5.2)	0.11	0.47	0.27	0.17	0.52
Berea sandstone	1891.6 (±13.1)	1813.9 (±15,3)	0.1384 (±0.0033)	1295.9 (±8.4)	849.4 (±11.0)	0.96	4.01	2.04	1.46	0.40

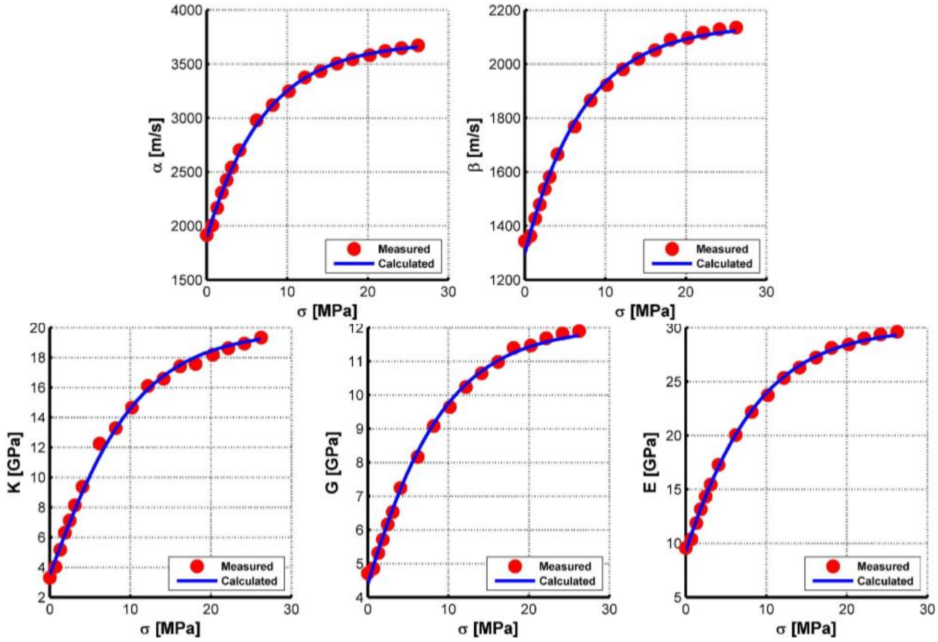

Figure 3 *Pressure dependent velocities and elastic moduli for the Berea sandstone*

Conclusions

The goal of our present investigation was to determine the pressure dependent elastic moduli for sandstones. They can be calculated if accurate P and S wave velocity data are available for any pressures. Therefore a rock physical model was developed which provides physical explanation for the pressure dependence of them. After estimating the model parameters by joint inversion and calculating the velocities, the pressure dependent elastic moduli can be derived as well. The accuracy of the inversion estimates and the reliability of the suggested petrophysical model was proved.

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