Geosciences and Engineering, Vol. 3, No. 5 (2014), pp. 145–153.

Statistical analysis of SHear strength parameters of municipal solid wastes by slope stability analysis

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ABSTRACT

The determination of the shear strength parameters of municipal solid waste material is one of the most difficult problems of the slope stability analysis of landfills. The aim of this study was to examine the frequencies and the correlation of the cohesion and the internal friction angle of wastes based on the collected results of previous international laboratory measurements, site investigations or back analysis.

INTRODUCTION

Geotechnical performance of a structure is strongly dependent on the properties of the constituent materials. These properties can be described using deterministic and/or probabilistic models. Deterministic models typically use a single discrete descriptor for the parameter of interest. Probabilistic models describe parameters by using discrete statistical descriptors or probability distribution (density) functions. The spatial variation of the properties can be described by using stochastic interpolation methods *(Jones et. al., 2002)*.

Uncertainty is usually our first thought, when trying to determine physical parameters for slope stability of landfills. Considering that the variability of the properties of wastes is much higher than of soils, we cannot solve geotechnical problems using the deterministic approach, because it cannot deal with this uncertainty. If we force the use of traditional methods our landfills will be either over-designed and not really cost effective, or unsafe and dangerous. Thus the importance of the application of probabilistic methods cannot be questioned in order to get more reliable results.

Variability of the properties of wastes is caused by different reasons, such as the circumstances and the technology of landfill construction, the age, degradation and composition of municipal waste, etc.

Several new geotechnical software packages provide the applicability of probabilistic methods in slope stability analysis, but for this we have to determine the frequencies and the correlation of the two shear strength parameters, of the cohesion and the internal friction angle of the waste material.

UNCERTAINTY IN GEOTECHNICS

Geotechnical variability results from different sources of uncertainties. The three primary sources are inherent variability, measurement error and model uncertainty, as described in *Figure 1.*

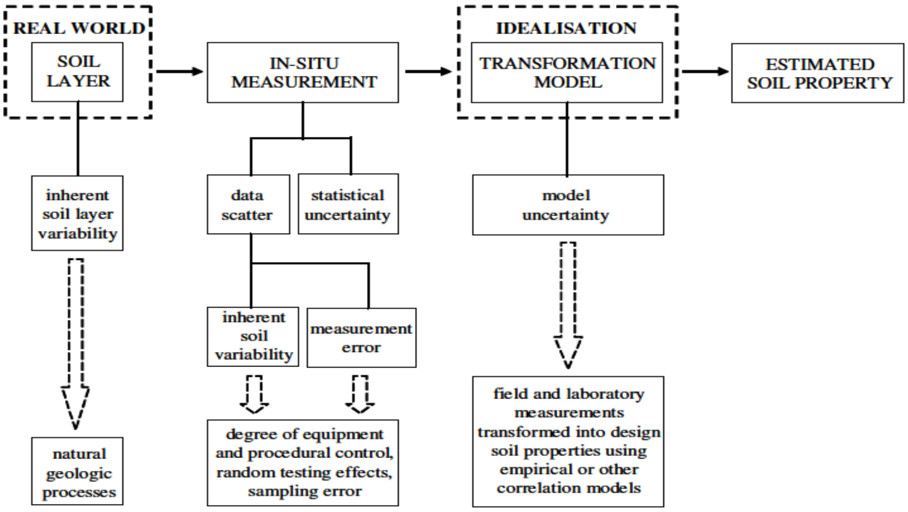


Figure 1. Uncertainty in soil property estimates (Kulhawy, 1992 in [RUSSELLI, 2008])

Inherent variability results primarily from natural geologic processes that created in-situ soil layers (or the variability of the collected waste material by landfilling). Measurement error is caused by sampling and laboratory testing.

This error is increased by statistical uncertainty that arises from limited amount of information. Finally the model uncertainty is introduced when field or laboratory measurements are transformed into input parameters for design models involving simplifications and idealizations *(Russelli, 2008)*.

The uncertainty in geotechnical properties of soils (and also waste materials) can be formally grouped into aleatory and epistemic uncertainty *(Lacasse et al., 1996 in [Jones et. al., 2002])*. Aleatory uncertainty represents the natural randomness of a property and, as such, is a function of the spatial variability of the property. Recognizing spatial variability is important because it can help distinguish the distances over which it occurs compared to the scale of the data of interest *(Whitman, 1996 in [Jones et. al., 2002])*. Epistemic uncertainty results from a lack of information and shortcomings in measurement and/or calculation. Epistemic uncertainty includes the systematic error resulting from factors such as the method of property measurement, the quantity of available data, and modeling errors.

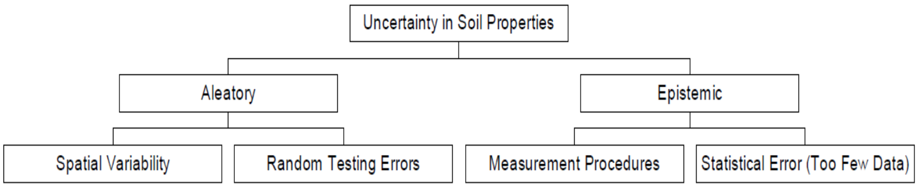


Figure 2. Sources of uncertainty in geotechnical soil properties (adapted from Whitman, 1996 in [JONES et. al., 2002])

Figure 2 illustrates the types of uncertainty in geotechnical soil properties. Human error would be considered a third source of uncertainty, however it is not considered in this overview because it is difficult to isolate and its effects on probability are usually included in compilations of statistics on aleatory uncertainty *(Jones et. al., 2002)*.

In a probabilistic analysis the geotechnical parameters, which represent the major sources of uncertainties, are treated as random variables. A random variable is a mathematical function defined on a sample space that assigns a probability to each possible event within the sample space.

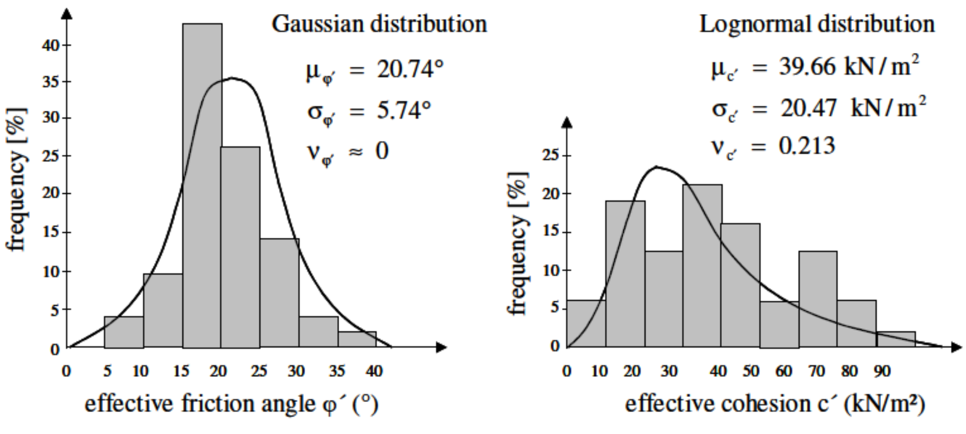
In practical terms, it is a variable for which the precise value (or range of values) cannot be predicted with certainty, but only with an associated probability, which describes the possible outcome of a particular experiment in terms of real numbers.

In this study the waste shears strength parameters: the cohesion and the internal friction angle are considered as random variables for the probabilistic analysis of the slope stability problem.

The most important statistical parameters related to the waste material properties variability are the mean value (μ_x), the standard deviation (σ_x), the skewness coefficient (ν_x) and the correlation coefficients (ρ_{xy}) between the two shear strength parameters. The variability of these data can be plotted graphically as histograms, or frequency diagrams *(Russelli, 2008)*.

The normal Gaussian distribution is the probability distribution most frequently used because of its symmetry and mathematical simplicity. It is commonly assumed to characterize many random variables where the coefficient of variation (COV = σ_x / μ_x)) is less than about 30%, as seen in *Figure 3 (a)* for the effective friction angle of the Frankfurt clay *(Russelli, 2008)*.

The lognormal distribution is generally accepted to reasonably model many soil properties, because it is strictly non-negative. It often provides a reasonable shape in cases where the coefficient of variation is larger than 30%, as for the effective cohesion of the Frankfurt clay in *Figure 3 (b)*. It can be concluded that the lognormal distribution may well represent the natural distribution for many spatially varying soil properties *(Russelli, 2008)*.



(a) (b)

Figure 3. Probability density functions of soil strength parameters for the Frankfurt clay (MOORMANN and KATZENBACH, 2000 in (RUSSELLI, 2008))

In geotechnical practice the normal Gaussian distribution was considered to characterize the shear strength parameters of soils where their correlation was supposed be not perfectly, but negative correlated and we presumed in previous works, that we can base our landfill slope stability research on experiences associated with soils. But the lack of exact knowledge can lead to fatal errors, thus the exact determination of the distribution of the shear strength parameters of wastes and their coefficient of correlation is essential.

SHEAR STRENGTH PARAMETERS OF WASTES

Shear strength parameters of different types of municipal solid wastes were measured, determined and discussed by several authors in geotechnical literature, but there are only some suggestions, how to take them into account by the slope stability analysis of landfills, such as *Manassero et al., (1996), König-Jessberger (1997), Sanchez-Alciturri et al. (1993)* and the ÖNORM (Austrian Standard) in *(Szabó, 2008)* and *(Szabó and Szabó, 2012)*.

Based on the collection of data in *(Szabó, 2008), (Varga, 2010, 2011a and 2011b)* and *(Szabó and Szabó, 2012)* we summarized the shear strength parameters in several tables and plotted graphically, see *Figure 4*.

As it can be seen in Figure 4a and b this collection was grouped based on the method of determination. Normal Gaussian fits of the relative frequencies of these data groups were graphed in Figure 5. and statistical parameters were determined (mostly by SPSS) in Table 1.

Both the graphs and the descriptive data in the table show, that the distribution of the internal friction angle of wastes is a little bit negatively skewed (compare with Figure 3 a, where the skewness of the density function of friction angle is 0, thus considered normal Gaussian distribution), with higher than 40% coefficient of variation. The distribution of the cohesion of wastes is a positively skewed, with higher than 80% coefficient of variation.

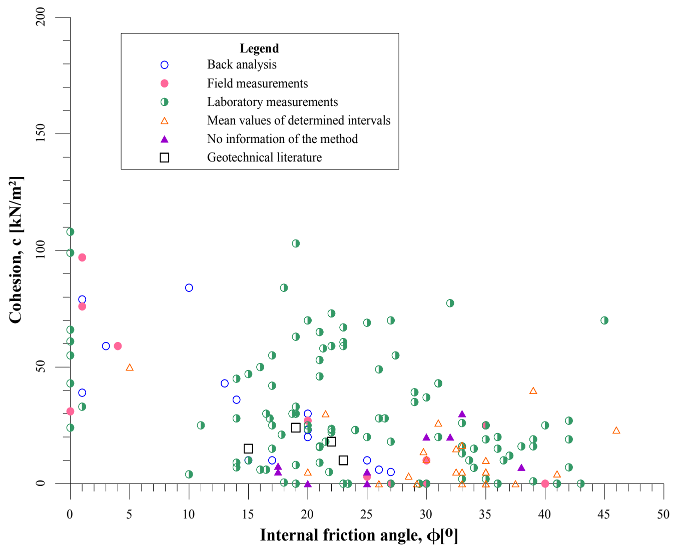
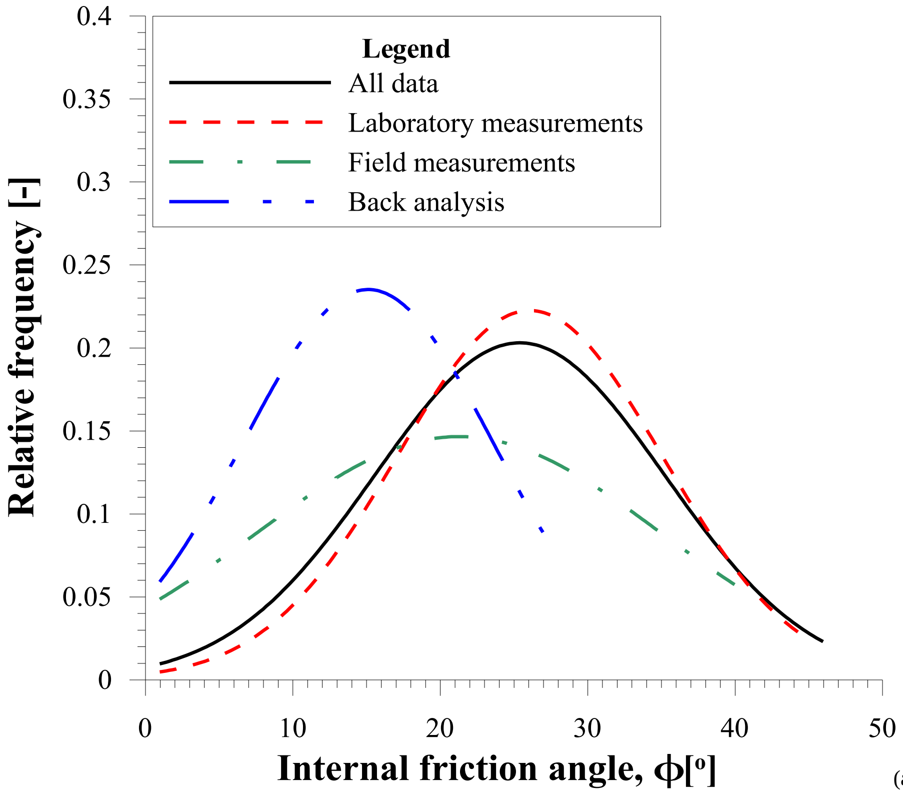


Figure 4. Shear strength parameters of wastes based on the results of back analysis, field and laboratory measurements and other geotechnical literature sources (Faur, 2012)

Some authors have collected information on the ranges of variation coefficient values for spatial variability of different soil properties, as derived from in-situ soil investigation and for the variability due to measurement errors. A well-known study on the COV values is that of *Phoon and Kulhawy (1999)*, which represents a good indication of the order of magnitude for the COV values of soil variability. In this study *Phoon and Kulhawy* presented data for sand and a clay layer and they found COV values between 5–15% for the effective friction angle. This range was also put forward by *Harr (1989)* and *Cherubini (1997)*. Occasionally higher COV values can be found for the friction angle, as in the report of Moormann and *Katzenbach (2000)*, where a value of about 30% is indicated for the Frankfurt clay. But this high COV value of the friction angle is not for a particular site, but for the entire Frankfurt area.



(a)

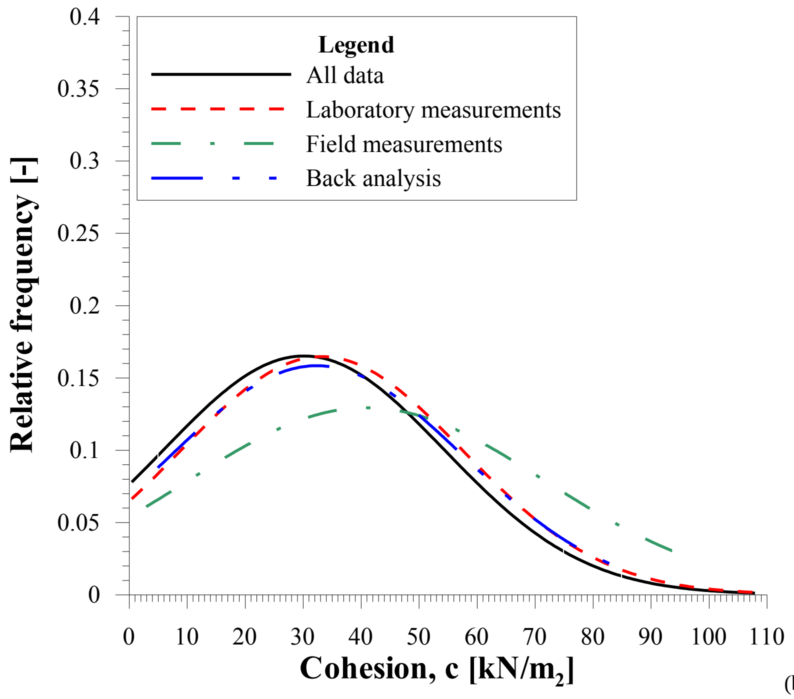


Figure 5. Normal Gaussian fits of the relative frequencies of these data groups (a) Internal friction angle (b) Cohesion

Table 1.a

Descriptives of the shear strength parameter data sets

	N [Number]	Minimum	Maximum	Mean (μ)	Standard Deviation (σ)
Valid N (listwise)	169	**All data [Unit in lines]**			
Cohesion, c [kN/m^2]	169	0.000	108.000	25.922	24.848
Internal friction angle, ϕ [°]	169	0.000	46.000	24.222	11.049
Valid N (listwise)	109	**Laboratory measurements [Unit in lines]**			
Cohesion, c [kN/m^2]	109	0.000	108.000	29.313	25.311
Internal friction angle, ϕ [°]	109	0.000	45.000	24.432	10.838
Valid N (listwise)	11	**Field measurements [Unit in lines]**			
Cohesion, c [kN/m^2]	11	0.000	97.000	29.818	33.630
Internal friction angle, ϕ [°]	11	0.000	40.000	19.364	15.075
Valid N (listwise)	14	**Back analysis [Unit in lines]**			
Cohesion, c [kN/m^2]	14	5.000	84.000	32.429	26.182
Internal friction angle, ϕ [°]	14	1.000	27.000	15.143	8.813

Table 1.b

Descriptives of the shear strength parameter data sets

	Coefficient of variation (COV) [%]	Variance	Skewness		Kurtosis	
		Statistic	Statistic	Std. Error	Statistic	Std. Error
Valid N (listwise)	**All data**					
Cohesion, c [kN/m²]	95.855	617.401	1.164	0.187	0.811	0.371
Internal friction angle, ϕ [°]	45.615	122.081	–0.451	0.187	–0.193	0.371
Valid N (listwise)	**Laboratory measurements**					
Cohesion, c [kN/m²]	86.348	640.653	0.979	0.231	0.469	0.459
Internal friction angle, ϕ [°]	44.359	117.461	–0.374	0.231	–0.069	0.459
Valid N (listwise)	**Field measurements**					
Cohesion, c [kN/m²]	112.783	1130.964	1.028	0.661	–0.055	1.279
Internal friction angle, ϕ [°]	77.852	227.255	–0.272	0.661	–1.700	1.279
Valid N (listwise)	**Back analysis**					
Cohesion, c [kN/m²]	80.737	685.495	0.914	0.597	–0.160	1.154
Internal friction angle, ϕ [°]	58.200	77.670	–0.438	0.597	–0.854	1.154

Considering the effective cohesion, *Harr* suggests in his work a value of about 20%. *Cherubini* presents values between 20–30%, *Li & Lumb (1987)* report a particular clay layer with a COV of 40% and *Moormann & Katzenbach* quote 50% for the Frankfurt clay. It can be seen that the available data on the effective cohesion show much more variation than for the effective internal friction angle (Russelli, 2008).

Table 2 shows the calculated correlations between the shear strength parameters.

Table 2

Correlations of the shear strength parameter data sets

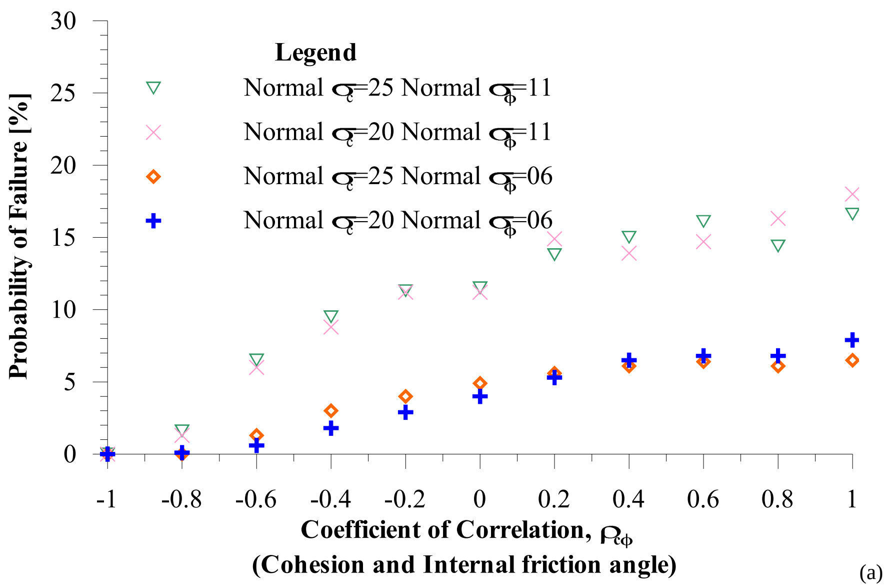
		All data		Laboratory meas.	
		Cohesion, c [kN/m²]	**Internal friction angle, ϕ [°]**	**Cohesion, c [kN/m²]**	**Internal friction angle, ϕ [°]**
Cohesion, c [kN/m²]	Pearson Correlation	1	–.487**	1	–.400**
	Sig. (2-tailed)		.000		.000
	N	169	169	109	109
Internal friction angle, ϕ [°]	Pearson Correlation	–.487**	1	–.400**	1
	Sig. (2-tailed)	.000		.000	
	N	169	169	109	109
		Field measurements		**Back analysis**	
		Cohesion c [kN/m²]	**Internal friction angle, ϕ [°]**	**Cohesion c [kN/m²]**	**Internal friction angle, ϕ [°]**

Cohesion, c [kN/m²]	Pearson Correlation	1	–.819**	1	–.785**
	Sig. (2-tailed)		.002		.001
	N	11	11	14	14
Internal friction angle, ϕ [°]	Pearson Correlation	–.819**	1	–.785**	1
	Sig. (2-tailed)	.002		.001	
	N	11	11	14	14
**. Correlation is significant at the 0.01 level (2-tailed).					

The correlation of the two shear strength parameters measured in laboratory is about –0.4, but the correlation of the results of field measurements and back analysis is about –0.8. Results of several running of slope stability analysis of a Hungarian landfill by Soilvision's SVSlope module are graphed on *Figure 6*.

CONCLUSIONS

The reduction of the standard deviation (SD) of the cohesion of caused only insignificant changes in the probability of failure, but the reduction of the SD of the internal friction angle caused remarkable changes in the probability of failure (see Figure 6 a). Thus if we decide to base our slope stability analysis on some own results instead of all the available data, it can lead to false conclusions.



(a)

Figure 6 b shows, that by negative correlations the effect of the application of the lognormal or the normal distributions for the shear strength parameters is insignificant, but if we decide to use the normal Gaussian distribution for the internal friction angle, and the lognormal distribution for the cohesion by slope stability analysis, we can stay on the safe side. As our results show, the correct determination of the statistical parameters of the shear strength parameters of municipal solid waste is essential to achieve reliable results of slope stability analysis of landfills.

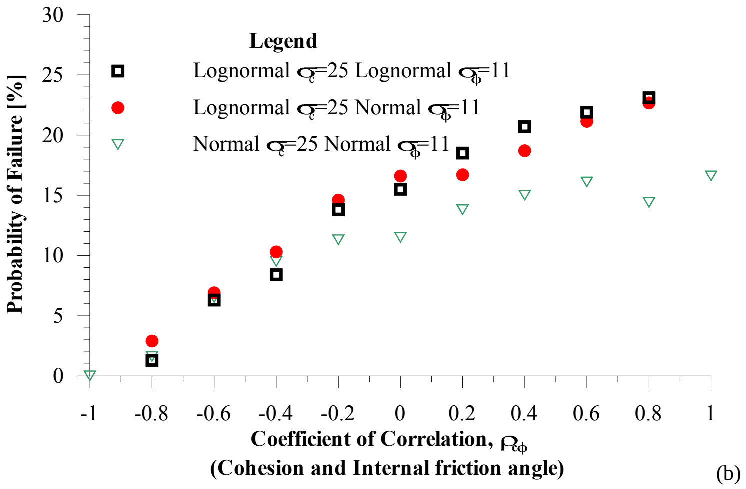


Figure 6.
Probability of failure by slope stability analysis as a function of the coefficient of correlation; (a) Effect of the reduction of the standard deviation (b) Effect of the application of the normal or lognormal distributions

ACKNOWLEDGEMENTS

I would like to thank Prof. Dr. Imre Szabó for his friendship and scientific support of all these years of research and co-work and wish him all the best to proceed.

The research was carried out in the framework of the Sustainable Resource Management Center of Excellence at the University of Miskolc, as part of the TÁMOP-4.2.2/A-11/1-KONV-2012-0049 “WELL aHEAD” project in the framework of the New Széchenyi Plan, funded by the European Union, co-financed by the European Social Fund.

REFERENCES

Faur K. B. (2012): *Hulladéklerakók magasítási lehetőségeinek ellenőrzése állékonyságvizsgálattal*. Záródolgozat. Budapest, SZIE YMÉK, 2012, p. 104.

Jones, A. L.–Kramer, S. L.–Arduino, P. (2002): *Estimation of Uncertainty in Geotechnical Properties for Performance-Based Earthquake Engineering*, University of Washington, 2002, p. 100.

Russelli, C. (2008): *Probabilistic Methods applied to the Bearing Capacity Problem*, Institut für Geotechnik der Universität Stuttgart, 2008, p.175.

Szabó I. (2008): *Hulladékelhelyezés* Egyetemi tankönyv Miskolci Egyetemi Kiadó, 1999, p. 440

Szabó I.–Szabó A. (2012): *Hulladéklerakók rekultivációja és utógondozása*, Miskolci Egyetem, p. 342.

Varga G. (2010): *Hulladéklerakók állékonysági kérdései* PhD értekezés Budapesti Műszaki és Gazdaságtudományi Egyetem 2010.

Varga G. (2011 a): Geotechnical Aspects of Bioreactor Landfilles. *Periodica Polytechnica. Ser.Civ.Eng*. Vol. 55, No.1. 2011. pp. 39–44. doi: 10.331/pp.ci. 2011–1.05

Varga G. (2011 b): Stability analysis of bioreactor landfills. *Proceedings of the XVth European Conference on Soil Mechanics & Geotechnical Engineering*, Athen, Septembre 2011. pp. 1827–1831. doi: 10.3233/978-1-60750-801-4-1827