TRIANGULAR AND SQUARE BRACED TUBULAR COLUMNS
Cost comparison of optimized column structures

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INTRODUCTION

Columns or masts are important parts of industrial buildings and sport stadiums**.** Various cross-sections can be used for them. Tubes are often used for their large stability. In the present study the column is constructed with three or four main circular hollow section (CHS) tubes and CHS bracings.
A 30 m high cantilever column is loaded by a compression force of F = 2x10^6 [N] and a horizontal force H =0.1F *(Figures 1-4)*. The aim of the present study is to compare the costs of the two different optimized structural versions. The advantage of triangular column is that it does not need transverse diaphragms to avoid torsional deformation of the cross-section.
The truss columns work as a built-up members, the effect of shear force during the buckling is considered according to Eurocode 3 [1]. The bracings are constructed from CHS as trusses with welded overlap K joints.
Unknowns to be optimized are as follows: CHS profile dimensions of the main tubes (chords) (D_0, t_0), tubular bracings (d_1, t_1) and diaphragms (d_2, t_2), distance between chords (h_1), number of spacings (q) (distances between K joints). The following constraints should be fulfilled: overall and local buckling of CHS chords and bracings, strength of overlap K joints according to ISO-IIW design rules for statically loaded hollow section joints (local yielding of overlapping brace, local chord yielding and brace shear).
Special fabrication constraints are formulated to make it possible the welding of joints: the angle between chords and bracing struts should be minimum 30^0 as well as the distance between strut ends in joints of chords, bracings and diaphragms should be minimum 2t (t is the thickness of bracings or diaphragms)
The cost function to be minimized contents the cost of material, cutting and grinding of CHS strut ends, welding of chords and joints as well as painting. The minima are found by a systematic search using a MathCAD algorithm. The cost comparison of the two optimized structural versions in a numerical problem shows that the triangular column is more economic than the square one. Although the dimensions of CHS struts are smaller for the square column, the bracing lengths are larger, thus the cost is also larger.

1 MINIMUM COST DESIGN OF A TRIANGULAR TUBULAR TRUSS COLUMN

Detailed optimization is treated for triangular column only.

1.1 Given data

$F, H, L, f_y, E, N = F/3$, $H = 0.1F$, $F = 2\text{x}10^6$ [N], L = 30 m, f_y = 355 MPa, $E = 2.1\text{x}10^5$ MPa

1.2 Variables

h_1 (distance between chords), D_0, t_0 (A_0, r_0, I_0 from table)(for CHS chords), d_1, t_1 (A_1, r_1 from table)(for CHS braces), $q=L/a$ number of segments (distances between welded joints in a chord).

1.3 Buckling constraint of a chord

Calculating the column as a built-up compression member according to Eurocode 3 [1]

$$\frac{N}{A_0}+\frac{Fa_0+HL}{W_y\left(1-\frac{F}{F_E}-\frac{F}{S_v\, cos\,30^0}\right)}\le \chi_0 f_y \qquad (1)$$

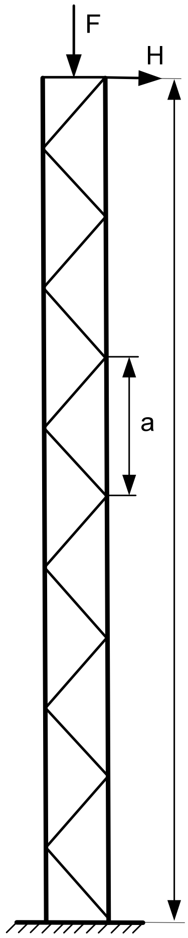

Fig.1. A braced tubular column

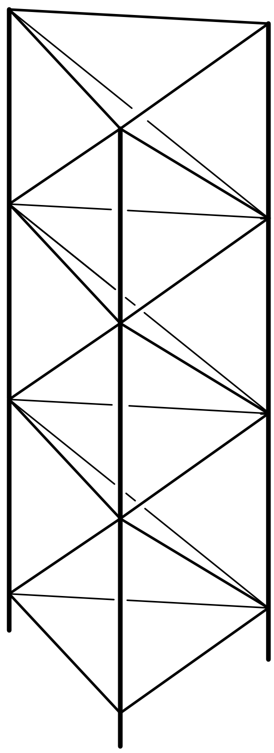

Fig.2. A part of the triangular tubular column

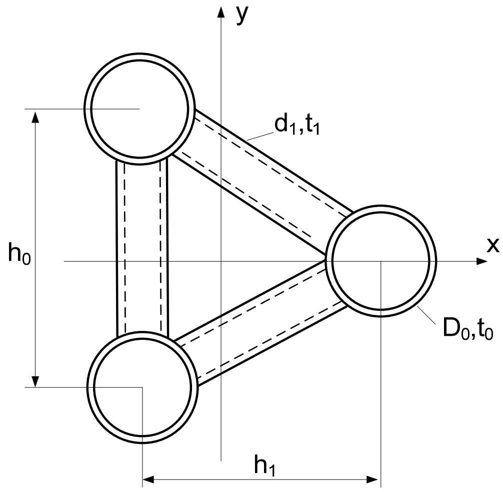

Fig.3. Cross section of the triangular tubular column

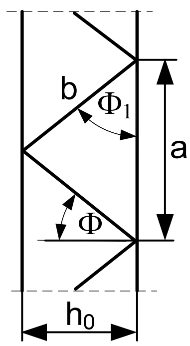

Fig.4. Geometric data of the bracing

$$a_0 = L/500 \qquad (2)$$

For a cantilever column the Euler buckling force $F_E=\frac{\pi^2 EI_{effy}}{4L^2}$ (3)

Effective moment of inertia

$$I_{effy}=3I_0+\frac{2}{3}A_0h_1^2 \qquad (4)$$

The section modulus

$$W_y=\frac{3I_{effy}}{2h_1} \qquad (5)$$

The coefficient of overall buckling according to Eurocode 3 [1]

$$\chi_0=\frac{1}{\phi_0+\sqrt{\phi_0^2-\bar{\lambda}_0^2}},\quad \bar{\lambda}_0=\frac{\lambda_0}{\lambda_E},\lambda_E=\pi\sqrt{\frac{E}{f_y}},\quad \lambda_0=0.9\frac{a}{r_0},\quad a=\frac{L}{q} \qquad (6)$$

$$\phi_0=\frac{1}{2}\left[1+\alpha_{01}\left(\bar{\lambda}_0-0.2\right)+\bar{\lambda}^2\right], \qquad (7)$$

Since α_0 =0.34 is for $a_0 = L/1000$, thus

$$\alpha_{01} = 2x0.34 = 0.68 \quad (8)$$

The factor considering the effect of shear during the overall buckling

$$S_v = \frac{EA_1 h_0^2 a}{b^3}, \quad h_0 = \frac{h_1}{cos\, 30^0}, \quad b = \sqrt{\frac{a^2}{4} + h_0^2} \quad (9)$$

1.4 Buckling constraint of a compression brace

$$\frac{N_d}{A_1} \le \chi_d f_y, \quad N_d = V_1 \frac{b}{h_0}, \quad V_1 = \frac{V}{2\, cos\, 30^0}, \quad (10)$$

for a cantilever column the shear force $V = \frac{\pi M_{Ed}}{2L}$ (11)

$$M_{Ed} = \frac{Fa_0 + HL}{1 - \frac{F}{F_E} - \frac{F}{S_v\, cos\, 30^0}} \quad (12)$$

The coefficient of overall buckling of a brace

$$\chi_d = \frac{1}{\phi_d + \sqrt{\phi_d^2 - \bar{\lambda}_d^2}}, \quad \phi_d = \frac{1}{2}\left[1 + 0.34\left(\bar{\lambda}_d - 0.2\right) + \bar{\lambda}_d^2\right], \quad \bar{\lambda}_d = \frac{\lambda_d}{\lambda_E}, \quad \lambda_d = 0.7\frac{b}{r_1} \quad (13)$$

1.5 Constraints on strength of overlap K joints of braces

It should be mentioned that the use of gap joints would be less economic than the overlap ones. Constraints according to Static design [2].

Local yielding of overlapping brace (Ov= 100%)

$$N_d \le N_{d\,max} = f_y t_1 s_{beff} \quad (14)$$

$$s_{beff} = \frac{\pi}{4}\left(2d_1 + 2d_{e.ov} - 4t_1\right), \quad d_{e.ov} = \frac{12}{d_1 / t_1} d_1 = 12t_1 \quad (15)$$

Local chord member yielding

$$\left(\frac{N_d}{N_{d.pl}}\right)^{1.7} + \frac{M_0}{M_{0.pl}} \le 1 \quad (16)$$

$$N_{d.pl} = A_d f_y \quad (17)$$

$$M_0 = H_d \frac{D_0}{2} = N_d \frac{a}{b}\frac{D_0}{2} \quad (18)$$

$$M_{0.pl} = W_{pl} f_y \quad (19)$$

Brace shear

$$N_s = 2N_d\, cos\,\phi_1 = 2N_d \frac{a}{2b} \le N_{s\,max} \quad (20)$$

$$N_{s\,max} = 0.58 f_u \frac{\pi}{4}\frac{\left(3d_1 + d_e\right)t_1}{sin\,\phi_1}, \quad f_u = 510 \text{ MPa}, \quad sin\,\phi_1 = \frac{h_0}{b} \quad (21)$$

$$d_e = \frac{12}{D_0 / t_0}\frac{t_0}{t_1} d_1 = \frac{12 t_0^2 d_1}{D_0 t_1} \quad (22)$$

1.6 Fabrication constraint to allow the welding of braces to chord

$$\left(d_1 + 2t_1\right) \le D_0 \frac{\pi}{6} \quad (23)$$

1.7 Cost function

The cost is calculated according to the fabrication sequence [3,4,5].

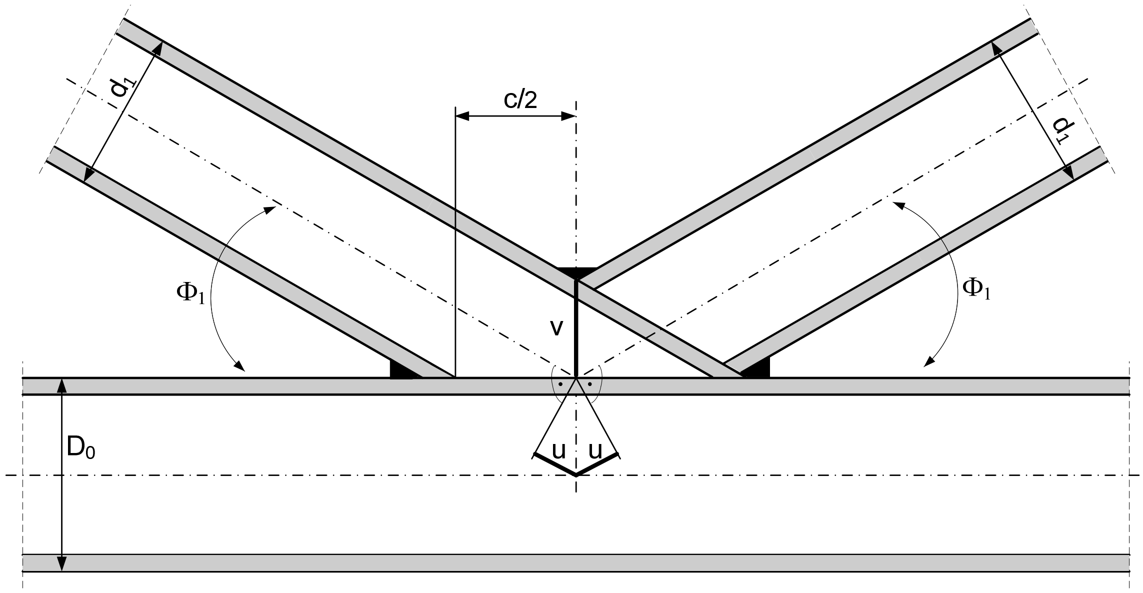

Fig. 5. Geometric data of the overlap K joint

According to *Figure 5*

$$u = \frac{D_0}{2} \sin\theta_1 = \frac{D_0 h_0}{2b} \qquad (24)$$

Length of the overlapped brace is

$$L_1 = b - 2u = b - D_0 \frac{h_0}{b} \qquad (25)$$

Length of the overlapping brace is calculated as

$$L_2 = b - 2u - \frac{2c}{4\cos\phi_1} = b - D_0 \frac{h_0}{b} - d_1 \frac{b^2}{a h_0} \qquad (26)$$

Cost of material

$$K_M = (k_{M0} V_0 + k_{M1} V_1)\rho,\ \rho = 7.85\text{x}10^{-6}\ \text{kg/mm}^3 \qquad (27)$$

$$V_0 = 3LA_0,\ V_1 = 3qA_1(L_1 + L_2) \qquad (28)$$

The k_M material cost factors for CHS according to Price list of the British Steel [6] are given in *Table 1.*

Table 1. Material cost factors

d (mm)	k_M (\$/kg)
88.9, 101.6, 114.3	1.0553
139.7, 168.3, 177.8, 193.7	1.1294
219.1, 244.5, 273.0, 323.9	1.2922
355.6, 406.4	1.3642
457.0, 508.0	1.4081

1.step: welding of the 3 main tubes.

It should be noted that the joints occasionally needed for transportation and assembly of parts of length 10 m or smaller are not treated.

(1a) Cost of cutting and grinding of CHS chord ends (together 30 ends)

$$K_{CG1} = \frac{2.5\pi D_0}{(350-2t_0)0.3} \quad (29)$$

(1b) Welding of chord elements of 2x5 m (together 9 elements) with butt welds

$$K_{W1} = k_W \left[\Theta_0\sqrt{4\rho V_{11}} + 1.3x0.152x10^{-3}t_0^{1.9358}D_0\pi\right], \quad (30)$$

$$V_{11} = A_0L_0, \; L_0 = 5000 \text{ mm.}, \; k_W = 1.0\$/\min, \; \Theta_0 = 2 \quad (31)$$

(1c) Welding of chords of total length with butt welds (together 3 chords)

$$K_{W11} = k_W \left(\Theta_0\sqrt{18\rho V_{11}} + 1.3x0.152x10^{-3}t_0^{1.9358}2D_0\pi\right) \quad (32)$$

2. step: welding of all the overlapped diagonals to the main tubes.

(2a) Cutting and grinding of overlapped CHS brace ends (total number of ends 6q)
According to *Figure 5*

$$c = \frac{d_1}{\sin\phi_1} = d_1\frac{b}{h_0} \quad (33)$$

$$K_{CG2} = \frac{2.5\pi d_1}{(350-2t_1)0.3}\frac{b}{h_0}6q \quad (34)$$

(2b) Welding of overlapped braces to the chords (total number of diagonals 3q)

$$K_{W2} = k_W\left(\Theta\sqrt{(3+3q)\rho V_2} + 1.3x0.7889x10^{-3}t_1^2 6q\pi d_1\frac{b}{h_0}\right), \; \Theta = 3 \quad (35)$$

$$V_2 = 3LA_0 + 3qA_1(b-2u) = 3LA_0 + 3qA_1L_1 \quad (36)$$

3.step: welding of all the overlapping diagonals to the previous structure.

(3a) Cutting and grinding of overlapping CHS grace ends (total number of ends 6q)

$$K_{CG3} = \frac{2.5\pi d_1}{(350-2t_1)0.3}\frac{b}{h_0}6q \quad (37)$$

(3b) Welding of overlapping braces (total number of ends 6q)

$$K_{W3} = k_W\left(\Theta\sqrt{(1+3q)\rho V_3} + 1.3x0.7889x10^{-3}t_1^2 6q\pi d_1\frac{b}{h_0}\right), \; \Theta = 3 \quad (38)$$

where $V_3 = V_2 + 3qA_1L_2$, (39)

Cost of painting

$$K_P = k_PS, \; k_P = 14.4\text{x}10^{-6} \; \$/\text{mm}^2 \quad (40)$$

The surface to be painted

$$S = 3L\pi D_0 + 3q(L_1 + L_2)\pi d_1 \quad (41)$$

The total cost

$$K = K_M + 30K_{CG1} + 9K_{W1} + 3K_{W11} + K_{CG2} + K_{CG3} + K_{W2} + K_{W3} + K_P \quad (42)$$

2 OPTIMIZATION RESULTS

The optimization is carried out by a systematic search using a MathCAD program. The CHS profile thicknesses are selected taken into account the local buckling constraint $d/t \le 50$. The cost is

calculated for selected chord and brace CHS profiles and for q = 3 -6 values, after the determination of h_1 considering the chord buckling constraint. The brace profile is checked for buckling. The welded overlap K-joints are checked for constraints given by Eqs.(14,16,20). The optimization results are given in *Table 2*.
Data of the optimum structure: $q = 5$, $h_1 = 4760$ mm, $D_0 x t_0 = 323.9x8$, $d_1 x t_1 = 139.7x4$.
Check of the constraints: (1): 251.8<252.0 MPa, (10): 128<170 MPa, (14): $2.19x10^5<4.01x10^5$ N, (16): 0.678<1, (20): $2.1x10^5<5.3x10^5$ N, (23): 148<170 mm, OK.

Table 2. Cost in $ for different chord profiles and q-values. Optimum is marked by bold letters, optima for another chord profiles are given in italics.

Chord profile	q	3	4	5	6
406.4x10	K	20560	*20390*	20760	21100
355.6x8	K	18330	17390	*17030*	17450
323.9x8	K	unreal	17080	**16510**	16910

The brace profile is 139.7x4 except for 406.4x10 as well as 323.9x8 and q = 3. Note that the structure for chord profile of 323.9x8 and q = 3, as well as chord profile of 273.0x6 cannot be realized
It can be seen that the cost increases when the chord profile increases, thus, larger chord profiles do not give cost minimum. All the optima fall in the range of q = 4-5, thus, it is enough to investigate values in the range of q = 3 -6.
The optimization of the square tubular column is performed similarly. The result is K_{min} = 19840$, which is (19880-16510)/198.8 = 17% larger than that for triangular column.
It can be concluded that the triangular tubular column is more economic, since the difference of cost minima is 17%.

3 ACKNOWLEDGMENT

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REFERENCES

[1] Eurocode 3. EN 1993-1-1: 2009. *Design of steel structures.* Part 1-1: General structural rules. Brussels, 2009.
[2] Static design procedure for welded hollow section joints – Recommendations. ISO 14346, IIW-doc. XV-1402-12. 2012.
[3] Jármai K., Farkas J., 1999. "Cost calculation and optimization of welded steel structures". *J Constr Steel Res* Vol.50, pp. 115-135.
[4] Farkas J., Jármai K., 2008. *Design and optimization of metal structures.* Chichester, UK, Horwood.
[5] Farkas J., Jármai K., 2013. *Optimum design of steel structures.* Heidelberg, etc. Springer.
[6] Price List 20. *Steel tubes, pipes and hollow sections.* Part 1b. Structural hollow sections. British Steel Tubes and Pipes, 1995.