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EXPLANATION OF PRESSURE DEPENDENCE OF ACOUSTIC VELOCITY BASED ON THE CHANGE OF PORE VOLUME

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ABSTRACT

Pressure strongly influences the mechanical and transport properties of rocks, such as acoustic velocity, porosity, permeability and resistivity. Seismic and borehole logging techniques measure these rock properties in order to infer subsurface information. To relate changes in seismic attributes to reservoir conditions, a thorough understanding of pressure effects on rock properties is essential. Therefore it is important to develop a petrophysical model based on simple physical assumptions which describes the relationship between acoustic velocity and pressure. The suggested model is based on the idea that the pore volume of a rock is decreasing with increasing pressure. The model was applied to acoustic P wave velocity data sets including measurement data published in literature by Xu et al. and Toksöz et al. The model parameters were determined by inversion method. The inversion results proved that the proposed petrophysical model performs well in practice.

INTRODUCTION

Propagation characteristics of acoustic wave carry information of important mechanical properties of rocks hence the determination of velocity is a common task in studying rock parameters both in laboratory and in-situ. The velocity of acoustic waves propagating in different rocks under various confining pressure values [8], [12], [16] and also under different pore pressures [4], [6], [10], [18] were investigated by many researchers. The phenomenon that the observable wave velocity is increasing because of increasing pressure is well-known and was explained on various rock mechanical studies [3], [11], [18]. One of the most frequently used mechanisms for explaining the phenomenon is based on the change of pore volume under pressure [3].

General observation is that the velocity of acoustic wave propagating in rocks is in non-linear connection with the effective pressure [2], [18]. The pressure-acoustic velocity connection can be characterized best by exponential function [11], [14–15]. Several empirical models exist to describe the pressure dependence of longitudinal acoustic wave, but these models usually provide the determination of the parameters of a suitably chosen formula based on mathematical regression method remaining the physical meaning unexplained [7], [15]. To reasonably interpret laboratory measurements, a quantitative model – which provides the physical explanation – of the mechanism of pressure dependence is required which includes as few parameters as possible. In the paper a petrophysical model for the description of the pressure dependence of propagation velocity is presented.

MEASUREMENT OF ACOUSTIC WAVE VELOCITY

The pulse transmission technique was used for P wave velocity measurements [13]. The measurements required special measuring equipment (Figure 1) which was compiled at the Department of Geophysics (University of Miskolc). The pulse generator emitted a pulse to the piezoelectric transmitter crystal which started an acoustic wave in the sample. The receiver crystal transformed the acoustic sign to electric pulse which was amplified. Another

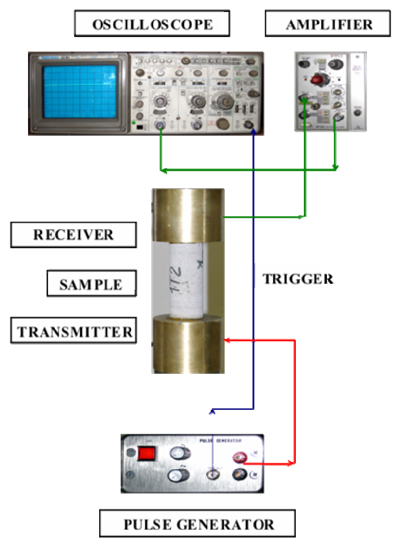

Figure 1. Measurement layout

pulse – the trigger – was emitted also by the pulse generator which synchronized the measurement system and the high-frequency digital oscilloscope which detected the arrival time of waves. In this way we can measure the propagation time of waves. Propagation velocity of acoustic waves can be determined by means of that of travel times and length of sample.

The Department performed wave velocity measurements on several sandstone samples originated from oil drilling wells. Two typical test results (fine-grained sandstones) are presented in the paper: Samples A and B. Rock samples subjected to uniaxial stress were analyzed by an electromechanical pressing device and wave velocities – as a function of pressure – were measured at adjoining pressures (up to ~15 MPa). The measured longitudinal velocity versus uniaxial pressure of the previously mentioned samples is shown in *Figure 2*. Measurement data indicate that the velocity increases first strongly nonlinearly with increasing pressure (because the quantity of pores are relatively high in this region) then in the higher pressure domain the increase in velocity (with increasing pressure) is moderate which can be attributed to the decrease of pore volume of rock sample, i.e. the pores are closing with pressure.

To confirm the reliability of the model independent data sets chosen from literature were processed as well. Xu et al. [17] and Toksöz et al. [13] also applied the pulse transmission

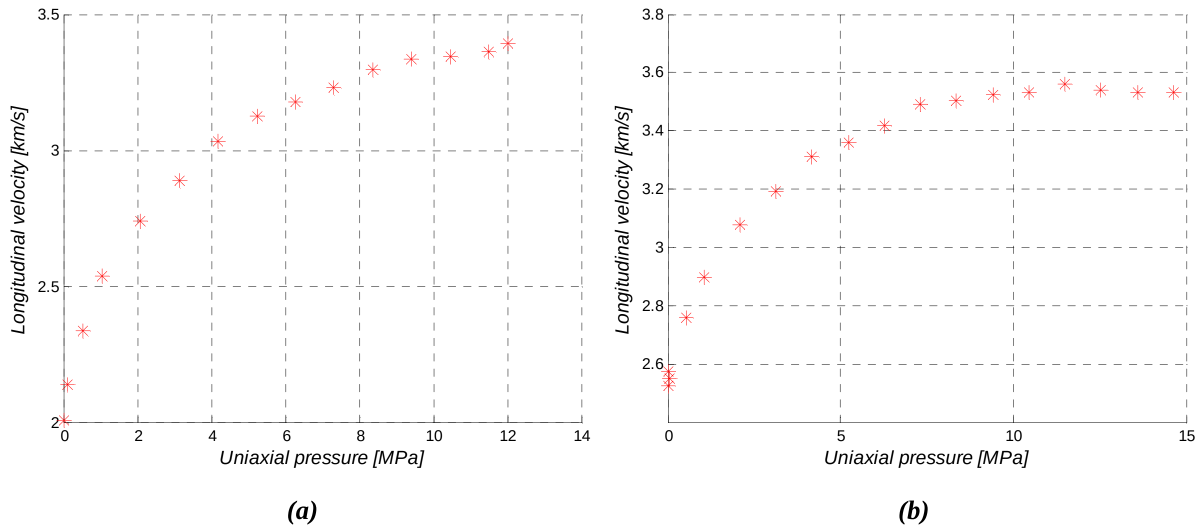

Figure 2.
Longitudinal velocity vs. uniaxial pressure of Sample A (a) and Sample B (b)

technique for velocity measurements. Each measurement was carried out at various pressures up to 70/35 MPa. The sample used by Xu et al. was low-porosity Lyons sandstone, a Permian aeolian deposit composed of mostly well-sorted quartz grains (90%) with less than 3% of clay. The Lyons sandstone is composed of rounded grains with a grain size of about 0.2 mm and is well cemented. Toksöz et al. analyzed a Berea medium-grained sandstone sample which was composed of angular grains showing microcracks and the grain contacts were somewhat jagged and were weakly cemented. It had an average porosity of 16%, permeability of 75 mD, and a bulk density of 2,61 g/cm^3.

THE PRESSURE DEPENDENT VELOCITY MODEL

The response of rock to stress depends on its microstructure, constituent minerals and porosity, which is manifested in pressure dependence of velocity of elastic waves. Following Birch's [3] qualitative considerations we assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure – from the unloaded state –, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, all pores are closed. Therefore we introduce the parameter *V* as the pore volume (per unit volume) of a rock. The model is restricted only for uniaxial stress state and longitudinal acoustic waves.

If a stress increase $d\sigma$ is created in a rock let us assume - because of the closure of pores - that the change of pore volume dV is directly proportional to the applied stress increase $d\sigma$ and also the pore volume V. One can describe the two assumptions with the following differential equation

$$dV = -\lambda_V V \, d\sigma \ , \tag{1}$$

where λ_V is new (positive) material quality dependent petrophysical constant. The negative sign represents that with increasing stress the pore volume decreases.
The solution of Eq. (1) is

$$V = V_0 \exp\left(-\lambda_V \sigma\right) \ , \tag{2}$$

where V_0 is the pore volume at stress-free state ($\sigma = 0$). We assume also a linear relationship between the infinitesimal change of the propagation velocity dv – due to stress increase – and dV

$$dv = -\alpha_V \, dV \ , \tag{3}$$

where α_V is a proportionality factor. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining this assumption with Eqs. (1–2) one can obtain

$$dv = \alpha_V \lambda_V V_0 \exp(-\lambda_V \sigma) \, d\sigma \tag{4}$$

and after integration

$$v = K - \alpha_V V_0 \exp(-\lambda_V \sigma) \ . \tag{5}$$

At stress-free state ($\sigma = 0$) the propagation velocity v_0 can be measured which is computed from Eq. (5) as $v_0 = K - \alpha_V V_0$. With this result and introducing the notation $\Delta v_0 = \alpha_V V_0$ Eq. (5) can be rewritten in the following form

$$v = v_0 + \Delta v_0 \left(1 - \exp(-\lambda_V \sigma)\right) \ . \tag{6}$$

Eq. (6) provides a theoretical connection between the propagation velocity and rock pressure. In the framework of the model, the velocity of acoustic wave increases from v_0 (at zero pressure) to $v_{max} = v_0 + \Delta v_0$ (at high pressure, when all the pores are closed). So, Δv_0 can be considered the velocity-drop caused by the presence of pores at zero pressure [7]. Note that in the range of high pressures, reaching a critical pressure [1] the reversible range is exceeded and destruction of the sample may occur and decreasing velocity can be observed.

This effect is outside of the author's present investigations. Therefore this model is valid only in the reversible range.

As it was mentioned λ_V is a new petrophysical constant, of which physical meaning is necessary to be given [5]. Introducing the notation $\Delta v = v_{max} - v$, (the velocity-drop caused by the presence of pores at pressure σ) Eq. (6) can be written in the form

$$\Delta v = \Delta v_0 \exp\left(-\lambda_V \sigma\right) \ . \tag{7}$$

Experiences denote that rocks show different velocity response to the same change in the rock pressure or in other words the velocity shows different sensitivity to pressure. It is interesting to see what amount of (relative) velocity change can be measured as a consequence of a certain (for example unit) change in the stress. For similar purpose, the sensitivity functions are extensively used in the seismic, geoelectric, electromagnetic and well-logging literature. Hence the author introduces the (logarithmic) stress sensitivity of the velocity-drop $\Delta v = v_{max} - v$ as

$$S\left(\sigma\right) = -\frac{1}{\Delta v}\frac{d\Delta v}{d\sigma} = -\frac{d\ln\left(\Delta v\right)}{d\sigma}. \tag{8}$$

Using Eq. (7) it can be seen that

$$\lambda_V = -\frac{d\ln\left(\Delta v\right)}{d\sigma} = S, \tag{9}$$

which shows that the petrophysical characteristic λ_V is the logarithmic stress sensitivity of the velocity-drop.

CASE STUDY

In order to prove the validity and practical applicability of the introduced petrophysical model, it was tested on longitudinal wave velocity data sets. The petrophysical constants (v_0, Δv_0, λ_V) appearing in the model equation (Eq. [6]) can be determined by processing measurement data based on the method of geophysical inversion. Linearized inversion method was used (principle of least squares method [9]). The inversion results for each sample can be seen in *Table 1*. The estimation errors – which are in parenthesis after each parameter – of the model parameters were calculated using the method given by Menke [9]. According to the method the elements of the main diagonal of covariance matrix in parameter space (*cov(m)*) provide the variances (σ_m) of model parameters, that means

$$\sigma_{m_i} = \sqrt{\operatorname{cov}\left(\mathbf{m}\right)_{ii}} \tag{10}$$

gives the estimated error of the i-th model parameter (i = 1,2,3 in the given problem).

Table 1

Model parameters estimated by linearized inversion

Sample	v_0 (km/s)	Δv_0 (km/s)	λ_V (1/MPa)
A	2,09 (±0,0141)	1,29 (±0,0163)	0,3229 (±0,0146)
B	2,56 (±0,0072)	0,99 (±0,0086)	0,3467 (±0,0093)
Lyons (Xu et al. 2006)	3,75 (±0,0108)	1,03 (±0,0119)	0,0611 (±0,0031)
Berea (Toksöz et al. 1979)	3,32 (±0,0098)	0,82 (±0,0108)	0,1330 (±0,0068)

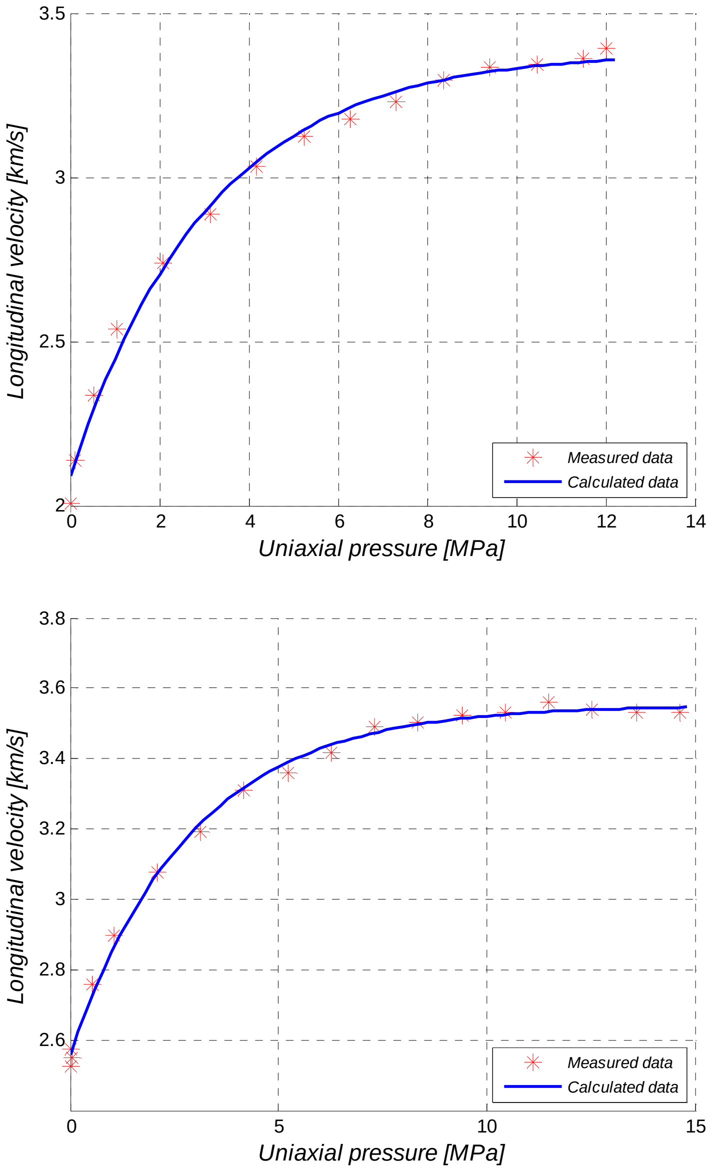

Figure 3.
Velocity-pressure function of sample A and B

With the estimated parameters, the velocities can be calculated at any pressure by substituting them into the model equation (Eq. [6]). The results are shown in *Figure 3–4*. The solid line shows the calculated velocity-pressure function while asterisk symbols represent the measured data. The figures show that the calculated curves are in good accordance with the measured data which proves that the petrophysical model applies well in practice. For the characterization of the accuracy of inversion estimates the author calculated the RMS (Root Mean Square) value according to the following formula [9]

$$D = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(d_k^{(m)} - d_k^{(c)}\right)^2} \cdot 100\,[\%], \tag{11}$$

where $d_k^{(m)}$ is the measured velocity at the k-th pressure and $d_k^{(c)}$ is the k-th calculated velocity data which can be computed by Eq. (6). To characterize the reliability of the suggested petrophysical model the mean spread was also calculated by [9]

$$S = \sqrt{\frac{1}{M(M\text{-}1)} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\mathrm{corr}(\mathbf{m})_{ij} - \delta_{ij} \right)^2}, \tag{12}$$

where δ is a Kronecker-delta symbol (which equals 1 if i = j, otherwise 0), *M* is the number of model parameters and *corr(m)* is the correlation matrix. Table 2 contains the calculated RMS and mean spread values for each sample in the last iteration step.

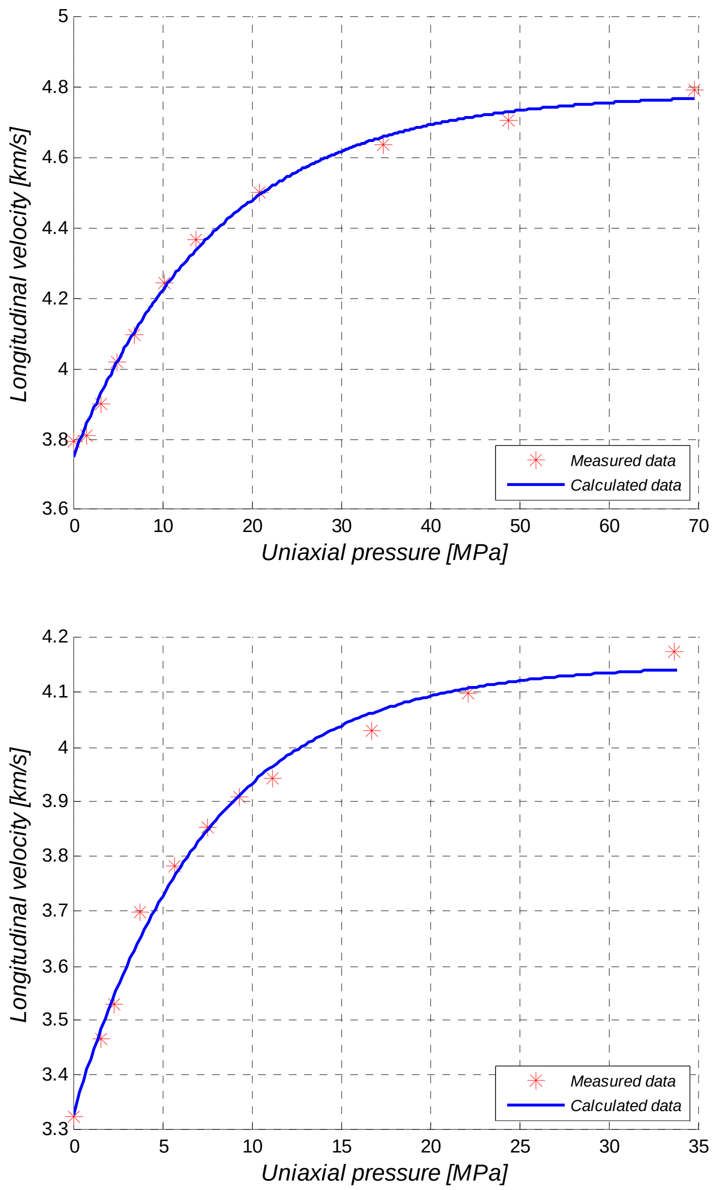

Figure 4.
Velocity-pressure function of Lyons and Berea sandstone samples

Table 2

Estimated RMS and mean spread values

Sample	D (%)	S
A	0,0369	0,4143
B	0,0209	0,4160
Lyons (Xu et al. 2006)	0,0247	0,4293
Berea (Toksöz et al. 1979)	0,0231	0,4433

It can be seen that the data misfits (RMS) were small (less than 1%), and the mean spread values indicate that the parameters are in moderate correlation, but the inversion results are still reliable. These results confirm the accuracy of the inversion estimates and the feasibility of the developed petrophysical model.

CONCLUSIONS

A new petrophysical model for describing the connection between the propagation velocity of acoustic wave and rock pressure was presented. The author found that the pressure dependence of acoustic velocity can be well described by a three-parameter exponential equation which gives also the physical explanation of pressure dependence: $v = v_0 + \Delta v_0\ (1\text{-}\exp(-\lambda_V \sigma))$, where v_0 is the velocity at zero pressure, Δv_0 is the velocity drop caused by the presence of pores and λ_V is a new petrophysical parameter. The physical explanation of each parameter is clarified in the paper.

Acoustic velocity measurement data of four different rock samples (two of them were chosen from the literature) were used to confirm the reliability of the model. By means of inversion-based processing the model parameters were determined from measurement data, thus calculated data could be produced by using the petrophysical model. The calculated data matched accurately with measured data proving that the petrophysical model performs well in practice. Inversion results confirmed the accuracy and feasibility of the petrophysical model. The model was also applied on several sandstone samples (fine-, medium-, coarse-grained, pebbly, tuffy etc.) with success during the research.

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LIST OF SYMBOLS

Symbol	Description	Unit
cov(m)	covariance matrix in parameter space	-
D	Root Mean Square (RMS)	%
$d_k^{(c)}$	calculated velocity at the k-th pressure value	km/s
$d_k^{(m)}$	measured velocity at the k-th pressure value	km/s
dv	change of propagation velocity	km/s
dV	change of pore volume (per unit volume)	-
$d\sigma$	stress increase	MPa
K	integration constant	-
M	number of model parameters	-

N	number of data	-
S	mean spread	-
$S(\sigma)$	stress sensitivity of velocity-drop	-
v	propagation velocity	km/s
V	pore volume (per unit volume)	-
v_0	propagation velocity at stress-free state	km/s
V_0	pore volume at stress-free state (per unit volume)	-
v_{max}	propagation velocity at maximum pressure	km/s
α_V	proportionality factor	-
δ	Kronecker-delta symbol	-
Δv	velocity-drop at pressure σ	km/s
Δv_0	velocity-drop	km/s
λ_V	new material quality dependent petrophysical constant	1/MPa
σ	stress	MPa
σ_{m_i}	estimated error of the i-th model parameter	-

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