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CHANGE OF TOOTH ROOT STRESS CALCULATION MODEL FOR NON-SYMMETRIC TOOTH SHAPE

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Abstract: In gear design the calculation of the strength has great important that national or international standards ordain. Models in these directives apply gears of symmetric tooth shape. In case of gears rotating in one direction asymmetric tooth shape arises. Model should be modified for calculations of non symmetric tooth shape.
Keywords: tooth root stress, non-symmetric teeth.

1. Introduction

In gear design many aspects have to be considered. For the calculation of the strength, the knowledge of the forces acting on the gear teeth, the possible failure forms of the machine elements and the material properties of the gear body are necessary depending on the type of the gear drive [2]. The basic standards for the sizing provide guidance in the gear design with symmetric teeth. Nowadays, the researchers have published several results on the possibility of the formation of asymmetric gears (see e.g., [6]). The need for this kind of change is primarily aimed by increasing the load capacity of the gears. Non-symmetric gear tooth shape can arise for power drives when gears are rotating in the same direction.

In this paper we are dealing with the determination of the stress at the tooth root for non-symmetric tooth shape for strength scaling.

2. Development of the strength scaling

The strength scaling processes has been changing significantly in the past two hundred years. In 1822, Tredgold was the first who introduced the strength calculations in sizing of the gears [2]. The attack of the force has been assumed on the edge of the head. This method has been further developed by Bach applying the idea that the body force was substituted by distributed force along the length of the tooth [1]. However, these calculation methods didn’t follow the exact shape of the tooth.

Later, with the expansion of knowledge and experience, the specific gear failures, which are primarily affected by the operating conditions, were also considered. Therefore, the stress at the tooth root, the surface pressure and the seizure were based in the sizing. These calculation principles were separated in time.

The earliest, the scaling for the tooth root capacity was spread. In 1893, Lewis developed a computational method that is taking into consideration the tooth shape [2]. The tooth, which was substituted by a parabola of uniform strength, was handled as a beam clamped at one end, i.e., a cantilever, and the loading force was assumed to be an evenly distributed force along the tooth length. His model is exhibited on Figure 1.

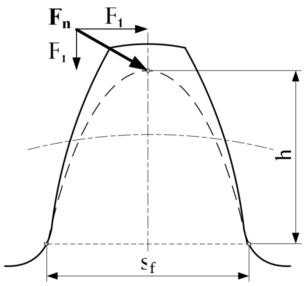

Figure 1. Lewis' model

For bending, the most dangerous section of the tooth is pointed by the point of the parabola which is tangent to the tooth root curve. The introduction of the notion of the tooth form factor is linked to Lewis.

Later Hofer refined Lewis' model. He marked out the dangerous section of the tooth root by straight lines angled 30° with the tooth centre line [3].

In 1908, Vidéky called the attention to the sizing for the surface pressure and through that for the influence of Hertz-stresses [1]. This method was further developed by Buckingham in 1926.

The seizure phenomenon called the researchers' attention to the warm-up conditions [2]. In this topic, Almen and Block got results in 1937. Dudley determined the required lifetime of gears through his calculations in 1954. Niemann developed a method in 1965, how to calculate with the actual operating conditions.

In 1950's, research works were published at both national and international levels on the gear strength scaling [1], [2]. The design recommendations of the American Gear Manufacturers Association (AGMA) have been published. Ten years later, in 1970, the national standard DIN3990 – in West Germany – and the international standard ISO 6336 were issued. Nowadays, the main regulations governing the calculation of the tooth root stress are summarized in ANSI/AGMA 2101-D04 (2004), DIN 3990-3 (1987), ISO 6336-3:2006 (see [5]).

3. Model creation

The models applied in strength calculations for the determination of the load capacity of the tooth are partially different. The difference is mostly due to the fact that which component of the load force on the tooth or normal tooth force is taken into account. The components of the normal tooth force, which is perpendicular to the toot surface, raise different stresses at the tooth root. The tangential component rises bending and shears at the tooth root and the radial component causes pressure on the root [1]. The difference between the calculations is due to the fact that which stress the calculation is performed on. The models contain simplifications of the real operating circumstances which are taken into consideration into the model with various modifying factors. Some of these factors depend on the geometry of the model.

For the calculation of the carrying capacity of the tooth root, one has to determine the nominal stress at the tooth root. The directives contain data of sizing for bending stress. The general form of the applied formula is given by expression (1), where σ_F describes the nominal stress at the tooth root, b denotes tooth width, F_t the tangential component of the normal tooth force, m the module and Y_F stands for the tooth form factor. The calculation is to be performed on both members of the gear pair.

$$\sigma_F = \frac{F_t}{bm} Y_F \cdot \qquad (1)$$

In each system, the determination of the tooth form factor depends on the type of the model. This means that the design methods can not be applied such that the tooth form factor is taken over from another method.

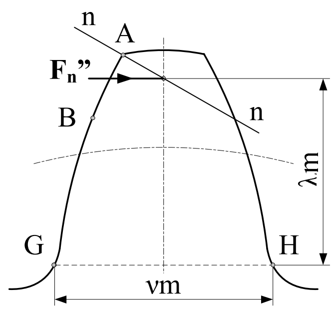

Figure 2. The calculation scheme of the load capacity of the tooth root when the bending is taken into consideration (Model 1)

On Figure 2, n-n denotes the line of action of the normal tooth force. Model 1 assumes that the tangential component of the normal tooth force load the tooth root is purely on bending and the tooth form factor is deduced from the bending stress written for the point G. This can be done when the normal tooth force is acting at an individual point (A) or at dual connection points (B). The values of ν and λ can be determined from the point set P_i (x_i, y_i) (i = 1, 2, …, n) of the base profile obtained by geometric mapping which is described by geometric conditions [8].

Model 2 is shown on Figure 3. The tooth form factor is deduced from the tooth root stress at the point G and it counts with both components of the normal tooth force.

On Figure 4, Model 3 considers the friction phenomenon at the contact points as well. At the entry of the contact, slipping occurs besides rolling (slipping with rolling). The resulting friction force together with the normal tooth force will change the load on the tooth. The tooth root stress, calculated at the point G will provide a different form as in Model 1 or in Model 2.

The calculation method of the tooth form factor should not be transferred from one system to another because the determining factors of the maximum value of the tooth root stress are also depend on the tooth shape and on the model.

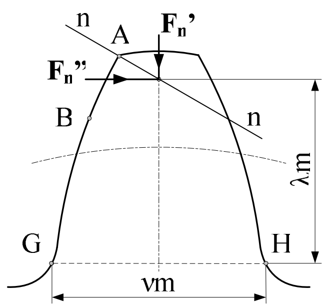

Figure 3. Both the bending and pressure are taken into consideration in the model of the load capacity for the tooth root (Model 2)

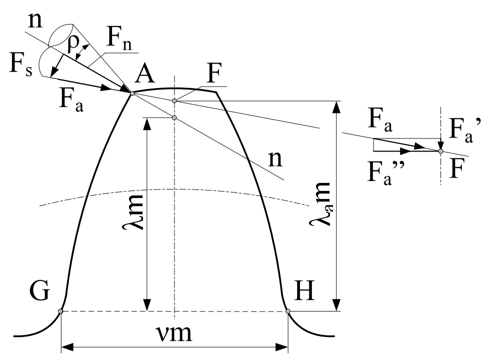

Figure 4. The load capacity model with the phenomena of bending, pressure and friction (Model 3)

4. Deviating from the standard

In case of gears rotating in the same direction, in a number of studies it was proved that the load capacity can be increased by the application of asymmetric toothed gears [7]. Di Francesco and Marini [7] have developed a computational method which allows the sizing of asymmetric gear teeth. Their concept is based on the ISO standard. The simplified view of their model is exhibited on Figure 5.

On the figure, the asymmetric tooth is denoted by letters HCAK'I', the left side of the tooth is the active (load transfer) side. The symmetric tooth is indicated by HCDKI. In the model, the cross section dangerous for bending (line HK') is located at the same distance from the gear centre as the cross section of the symmetric tooth (line HK) dangerous for bending. Line HK is appointed by the tangent lines which have angle 30° with the centre line of the symmetric tooth. Point L' bisects HK'. The axis of the asymmetric tooth passes through the point L'.

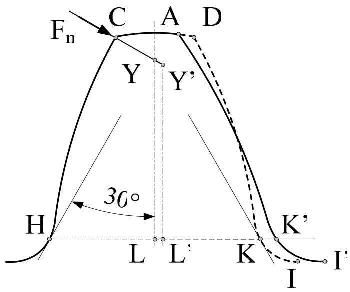

Figure 5. Asymmetric tooth model by Di Francesco and Marini

The point Y' on the axis of the asymmetric tooth is the intersection point of the axis and the line of action of the normal tooth force. The authors took the distance between the dangerous cross section and the force component causing bending at the tooth root with LY due to the small difference.

It should be noted that this approximation is not applicable for each model as the difference between LY and L'Y' is not negligible e.g., at the model which counts with the friction, the intersection point of the line of action of the force and the modified symmetry line can result significantly larger deviation.

Returning back to the model of [7], the authors have taken distance LY as the arm of the bending force. The difference comparing to the symmetrical tooth was resulted by the change of the tooth form factor and the stress concentration factor in the dangerous cross section (HK').

5. Deviating from the standard

If we modify the model then we have to change the tooth form factor at the same time. According to the normal tooth force loading to the tooth and the friction, the asymmetry of the tooth causes significant changes in the model.

The operational and supporting sides of the tooth are different which phenomenon is generated by the different profile angles of the basic profile and the different radii of the top roundings at the tooth root. As a result, the cross section of the tooth root will change, will increase, which is also modified by the number of teeth and the profile shift when we do not use uncorrected toothing.

The concept of the proper choice of the "new" dangerous cross section and the placement of the axis of this section is an important question.

Due to the changed geometry, an additional question arises: where the greatest stress occurs in the tooth root? The tangent line to the tooth root curve for involute derived by profile angle different than 20°, can be edited with another angle to the axis of the asymmetric tooth as in the symmetric case. These questions need to be answered after calculations.

Among the factors involved in the nominal tooth root stress, beside the tooth form factor the stress concentration factor depends on the geometrical shape of the model. This paper is not concerned with the latter.

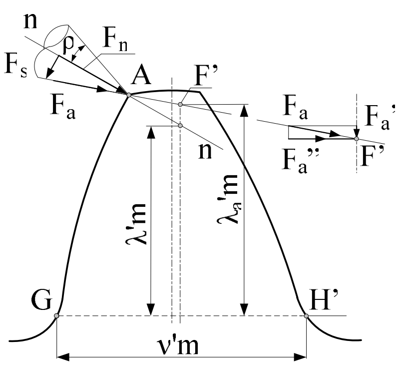

Figure 6. Computational model for asymmetric tooth

6. Summary

The standards of gear sizing are developed for the strength calculation of the symmetric tooth shape for specific model describing the geometry of the tooth. In case of power drives, the sizing is basically done for bearing capacity of the tooth root. A number of researches are published on the applicability of the asymmetric tooth gears from the point of increasing the capacity. Computational models have been proposed for the modifications which handle the asymmetry. In order to approximate the processes more accurately it is necessary to develop model which follows better the changes of the geometry and which takes into consideration the friction besides the components of the normal tooth force.

7. Acknowledgement

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