20th European Meeting of Environmental and Engineering Geophysics

Near Surface Geoscience 2014

Tu PA2 09

Proving the Applicability of the Petrophysical Model Describing Acoustic Hysteresis of P and S Wave Velocities

J. Somogyi Molnar* (MTA-ME Research Group of Geoeng., Univ. Miskolc), A. Kiss (University of Miskolc), H. Szegedi (University of Miskolc) & M. Dobroka (University of Miskolc)

SUMMARY

Due to the increasing demand for hydrocarbons and the depletion of the known hydrocarbon fields there is a growing claim to predict rock physical parameters more accurate at non-conventional conditions also. It is well known that acoustic velocity in rocks strongly depends on pressure which influences the mechanical, transport and elastic properties of rocks as well as wave propagation under pressure is very nonlinear and the quasistatic elastic properties of rocks are hysteretic. Characterization of hysteretic behavior is important for mechanical understanding of reservoirs during depletion. Therefore a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which provides the connection between the propagation velocity of acoustic waves (both P and S) and rock pressure both in case of pressurization and depressurization phases as well as explains the mechanism of acoustic hysteresis. The developed model is based on the idea that the pores in rocks close under loading and reopen during unloading. The model was applied with success to acoustic P and S wave velocity data sets.

Introduction

Geophysics has a wide palette to determine rock physical parameters, for example acoustic velocity, porosity, permeability, elastic moduli and it is well known that pressure has a strong influence on them. To interpret seismic, acoustic borehole logging data and to relate laboratory measurement to in-situ parameters properly it is necessary to understand what the effects of pressure are to them. Therefore the investigation of pressure dependence of propagation wave velocity in rocks is in focus of researchers. It is well known that the change of acoustic wave velocity propagating in rocks under pressure is highly nonlinear and the quasistatic elastic properties of rocks are hysteretic (Ji et al. 2007). The observable non-elastic response to pressure (namely acoustic hysteresis) may be caused by the irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions. The first theory assumes that the microcracks closed during pressurization do not reopen during subsequent depressurization (Walsh and Brace 1964). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original dimensions at lower pressures (Birch 1960). By idea of the improvement of contact conditions (Hashin and Shtrikman 1963) in a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

General observation is that the relationship between velocity and pressure is nonlinear in the beginning phase of loading and often can be described with exponential function. In literature several qualitative models are available to describe the pressure dependence of acoustic wave velocity but these empirical models do not explain the physical meaning of the process (Ji et al. 2007), they only give the regression function of the curve fitted to the measured data. To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper we present a quantitative petrophysical model, which explains the mechanism of pressure dependence of P and S wave velocity as well as describes well the acoustic hysteresis.

Describing acoustic hysteresis of P and S wave velocity

The response of rocks to stress depends on their microstructure, constituent minerals and porosity, which is manifested in pressure dependence of velocity of elastic waves. Following Birch's (1960) qualitative considerations we assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure -from the unloaded state-, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Therefore we introduce the parameter *V* as the unit pore volume of a rock. If a stress increase $d\sigma$ is created in a rock let us assume that the change of pore volume dV is directly proportional to the applied stress increase $d\sigma$ and also the pore volume *V*. One can describe the two assumptions with the following differential equation

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \tag{1}$$

where λ_V is new material quality dependent petrophysical parameter (Dobróka and Somogyi Molnár 2012) and V_0 is the pore volume at stress-free state ($\sigma = 0$). The negative sign represents that with increasing stress the pore volume decreases ($\lambda_V > 0$). We assume also a linear relationship between the infinitesimal change of the P wave propagation velocity $d\alpha$ - due to stress increase - and dV

$$d\alpha = -\kappa_P dV, \tag{2}$$

where κ_P is a positive proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining Eqs. (1-2) and solve the differential equation one can obtain

$$d\alpha = \kappa_P \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \rightarrow \alpha = K - \kappa_P V_0 \exp(-\lambda_V \sigma), \quad (3)$$

where K is an integration constant. At stress-free state (σ=0) the propagation velocity α_0 can be measured which is computed from Eq. (3) as $\alpha_0 = K - \kappa_P V_0$. With this result and introducing the notation $\Delta\alpha_0 = \kappa_P V_0$ Eq. (3) can be rewritten in the following form

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - \exp(-\lambda_V \sigma)). \quad (4)$$

Eq. (4) provides a theoretical connection between the propagation velocity and rock pressure in case of pressurization. Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded, hence decreasing velocity is observed. This effect is outside of our present investigations.

To characterize the depressurization phase, $v=V_0-V$ as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed pores start to open again, so decreasing velocity can be measured. Therefore we assume dv (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease $d\sigma$

$$dv = \lambda'_V v \, d\sigma \rightarrow v = v_m \exp\left(-\lambda'_V (\sigma_m - \sigma)\right), \quad (5)$$

where λ'_V is another new material characteristic constant and v_m is the closed pore volume at maximum pressure value σ_m. After Birch (1960) there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during pressurization do not reopen completely during depressurization. This irreversibility is denoted by two different parameters λ_V and λ'_V in our model. Combining Eq. (2) and Eq. (5) by using the formulas $dV=-dv$ and $\kappa_P v_m = \Delta\alpha_m$ one can find

$$\alpha = \alpha_m - \Delta\alpha_m \left(1 - \exp\left(-\lambda'_V (\sigma_m - \sigma)\right)\right). \quad (6)$$

Eq. (6) shows the propagation velocity – pressure function of depressurization phase. In the two limiting cases (at pressure value $\sigma=\sigma_m$ and $\sigma=0$) Eq. (6) gives α_m and $\alpha_1 = \alpha_m - \alpha v_m \left(1 - \exp\left(-\lambda'_V \sigma_m\right)\right)$ respectively, (here the notation $\alpha(0) = \alpha_1$ was used). This gives the formula (similar to Eq. (4))

$$\alpha = \alpha_1 + \Delta\alpha_1 (1 - \exp(-\lambda'_V \sigma)), \quad (7)$$

with the notation $\Delta\alpha_1 = -\alpha v_m \exp\left(-\lambda'_V \sigma_m\right)$.

Since the base of the model is the change of pore volume (which is independent of the direction of loading) it is valid in case of S waves. Following the same procedure similar model equations can be obtained e.g. for the pressurization phase the model contains parameters β_0, $\Delta\beta_0$, λ_V.

Case studies

To confirm the reliability of the model velocity data sets were processed. The pulse transmission technique was used for P wave velocity measurements. We performed measurements on many different sandstone samples which were subjected to uniaxial stresses by an automatic acoustic test system (Fig. 1a). It contains a load frame (max. 300KN) and a pressure generator (max. 80MPa). Wave velocities - as a function of pressure - were measured at adjoining pressures during pressurization and depressurization phases. To avoid the destruction of the samples we loaded them only up to the 1/3 of uniaxial strength. One typical test result (Sample 1: fine-grained sandstone) is presented in the paper. Our measurements showed that the longitudinal velocity is increasing with pressure. Moreover a slight difference between the characteristics of the pressurization and depressurization curves was found which can be explained by the phenomenon of acoustic hysteresis. At present we are not able to measure S wave velocity therefore we tested our theory on velocity data published in literature (Gomez et al. 2010). P and S wave velocity data measured by the pulse transmission technique on Oligocene Fountainebleau sandstone marked F410 (porosity was 6%) was processed.

Proving the validity and applicability of the introduced velocity model, we present the interpretation of measurement data of the described samples. The parameters appearing in the model equations (in case of pressurization and depressurization) valid also for P and S wave can be determined by processing measurement data based on joint inversion method (the Damped Least Squares Method). The inversion results for each sample can be seen in Table 1. For the characterization of the accuracy of inversion estimates the measure of fitting in data space (*D*) is also provided. It can be seen that the data misfits were small which confirm the feasibility of the developed petrophysical model.

Table 1 *Model parameters and data misfits of pressurization and depressurization phases estimated by joint inversion using the developed model.*

P wave							
Sample	**Pressurization**			**Depressurization**			D (%)
	α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ_V (1/MPa)	α_1 (km/s)	$\Delta\alpha_1$ (km/s)	λ'_V (1/MPa)	
1	1,92	1,22	0,10	1,99	1,19	0,26	0,86
F410	3,58	1,46	0,12	4,36	0,72	0,15	1,14
S wave							
Sample	**Pressurization**			**Depressurization**			D (%)
	β_0 (km/s)	$\Delta\beta_0$ (km/s)	λ_V (1/MPa)	β_1 (km/s)	$\Delta\beta_1$ (km/s)	λ'_V (1/MPa)	
F410	2,22	0,99	0,12	2,73	0,49	0,13	1,07

With the estimated parameters the velocities can be calculated (separately at pressurization and depressurization) at any pressure by substituting them into Eq. (4) or Eq. (7). The results are shown in Fig. 1b and Fig. 2, where the solid line shows the calculated velocity-pressure function produced by the velocity model, while symbols represent the measured data. The figures show that the calculated curves are in good accordance with the measured data proving that the petrophysical model describing acoustic hysteresis applies well in practice in case of P and S waves, too.

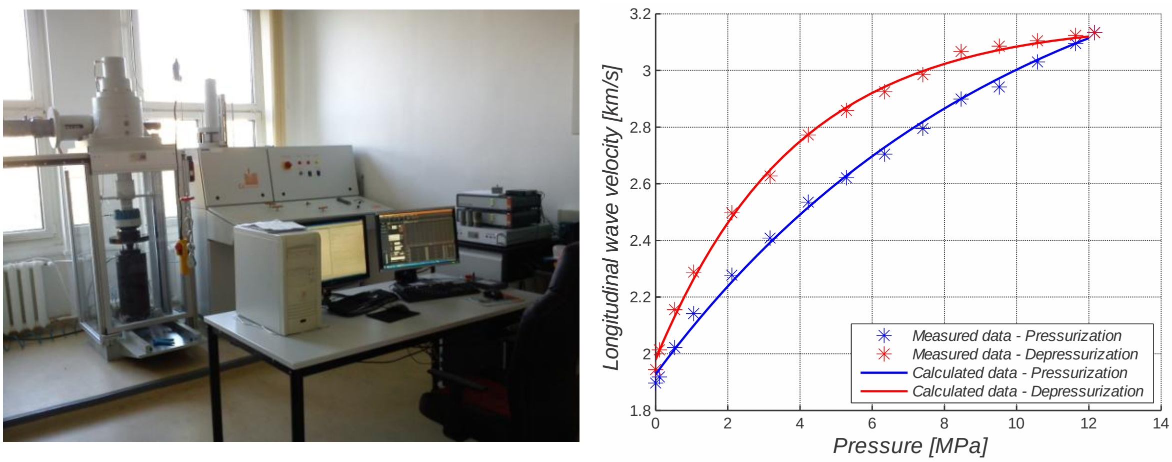

Figure 1 a.) *Automatic acoustic test system.* ***b.)*** *P wave velocity as a function of pressure of Sample 1 in case of pressurization and depressurization.*

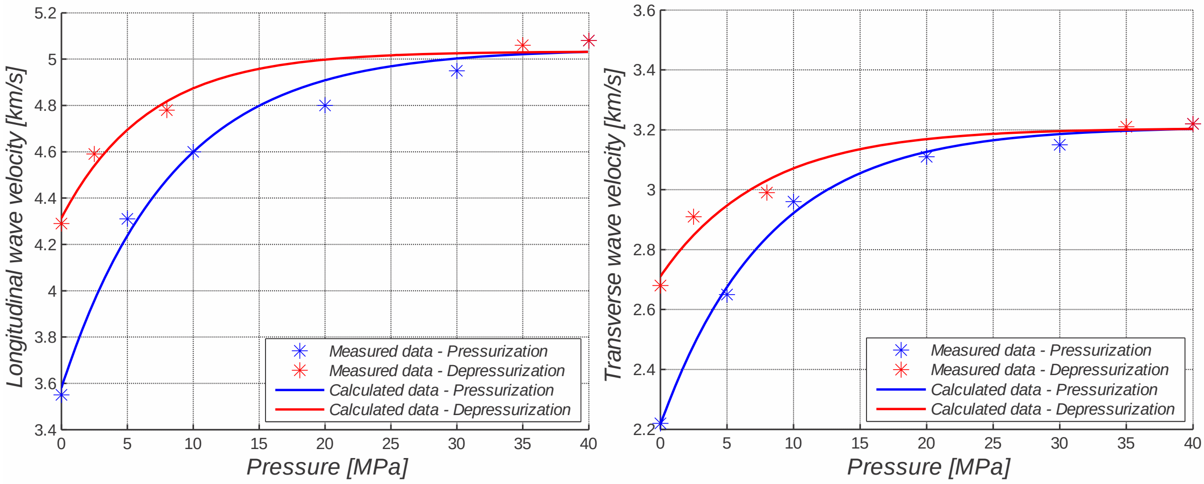

***Figure 2** P and S wave velocities as a function of pressure of Sample F410 in case of pressurization and depressurization. Data obtained from Gomez et al. (2010).*

Conclusions

In this paper based on Brace's theory, a petrophysical model describing acoustic hysteresis was presented. It provides the connection between the P and S wave velocity and rock pressure, both in case of pressurization and depressurization phases. The advance of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The model (valid only in reversible/elastic range) is based on the idea that pore volume changes with pressure. P and S wave velocity measurement data of sandstone samples were used to confirm the reliability of the model. By means of inversion-based processing the model parameters were determined from measurement data. Calculated data could be produced by using the petrophysical model and a very good fit between measured and calculated data was found thus inversion results confirmed the accuracy and feasibility of the petrophysical model.

Acknowledgements

The research of Judit Somogyi Molnár was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4.A/2-11-1-2012-0001 'National Excellence Program'. The research of Anett Kiss, Hajnalka Szegedi and Mihály Dobróka was supported by the OTKA project No. K 109441.

References

Birch, F. [1960] The velocity of compression waves in rocks to 10 kilobars, Part 1. *Journal of* Geophysics Research, **65**, 1083-1102.

Dobróka, M. and Somogyi Molnár, J. [2012] New petrophysical model describing the pressure dependence of seismic velocity. Acta Geophysica, **60**, 371-383.

Gomez, C.T, Dvorkin, J. and Vanorio, T. [2010] Laboratory measurements of porosity, permeability, resistivity, and velocity on Fontainebleau sandstones. Geophysics, **75**, E191-E204.

Hashin, Z. and Shtrikman, S. [1963] A variation approach to the theory of the elastic behaviour of multiphase materials. Journal of the Mechanics and Physics of Solids, **11**, 127– 140.

Ji, S., Wang, Q., Marcotte, D., Salisbury, M.H. and Xu, Z. [2007] P wave velocities, anisotropy and hysteresis in ultrahigh-pressure metamorphic rocks as a function of confining pressure. Journal of Geophysical Research, **112**, B09204.

Walsh, J.B. and Brace, W.F. [1964] A fracture criterion for brittle anisotropic rock. Journal of Geophysics Research, **69**, 3449-3456.