COMBINED GLOBAL/LINEAR INVERSION OF WELL-LOGGING DATA IN LAYER-WISE HOMOGENEOUS AND INHOMOGENEOUS MEDIA

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In the paper a combined inversion algorithm solving the nonlinear geophysical well-logging inverse problem is presented. We apply a successive combination of a float-encoded Genetic Algorithm as a global optimization method and the well-known linearized Marquardt algorithm forming a fast inversion procedure. The technique is able to decrease the CPU run time at least one order of magnitude compared to the Genetic Algorithm and gives the parameter estimation errors having a few linearized optimization steps at the end of the iteration process.

We use depth-dependent tool response equations to invert all the data of a greater depth-interval jointly in order to determine petrophysical parameters of homogeneous or inhomogeneous layers in one inversion procedure. The so-called interval inversion method gives more accurate and reliable estimation for the petrophysical model parameters than the conventional point by point inversion methods. It also enables us to determine the layer-thicknesses that can not be extracted from the data set by means of conventional inversion techniques. We test the combined interval inversion method on synthetic data, and employ it to the interpretation of well logs measured in a Hungarian hydrocarbon exploratory borehole.

Keywords: Genetic Algorithm; Marquardt algorithm; combined inversion; well-logging data; petrophysical parameters; interval inversion.

1. Introduction

In the industrial practice the well-logging inverse problem is usually solved by local gradient-based linearized point by point inversion methods. Having enough a priori information about the formations investigated they work as very quick and effective algorithms, but they tend to assign the solution to a local optimum of the objective function. As a rule this problem is treated by global optimization methods that search for the absolute extremum of the objective function. For example the Genetic Algorithm proposed by Holland (1975) is a very effective global optimization method, which solves the optimization process by the analogy with natural selection in biology. It gives always satisfactory results, but has got a strong disadvantage, that is the long computational runtime in particular.

The conventional point by point inversion methods are able to determine petrophysical parameters only to one depth-point. They can be improved, if we invert data from a greater depth-interval jointly. It is advantageous, because of the increased over determination, which leads to more accurate and reliable parameter estimation. For this purpose Dobróka (1995) introduced the so-called interval inversion algorithm. It is applicable to determine layer characteristic parameters and layer-thicknesses together in one inversion procedure. It is also possible to handle with more complicated petrophysical models by means of a series expansion of model parameters based on appropriately chosen basis functions.

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2. The inverse problem

In the zone of hydrocarbon-bearing formations we generally search for the value of such petrophysical parameters that underlie the calculation of reserves. In formulating the inverse problem, let us introduce the column vector of the model parameters of a shaly-sand sequence of strata in a given depth-point as

$$\vec{m} = [POR, SX0, SW, VSH, VSD]^T , \qquad (1)$$

where POR - effective porosity [fraction], SX0 - water saturation in the flushed zone [fraction], SW - water saturation in the virgin zone [fraction] , VSH - specific volume of clay [fraction], VSD - specific volume of sand [fraction].

In forward modeling an estimate for Eq.(1) is given and theoretical well-logging data are computed by means of given petrophysical relations. As a rule an appropriate well-logging measurement consists of lithological, porosity and saturation logs. Let us define the following vector that contains the measured data of a possible combination of well logs in a certain depth point

$$\vec{d}^{(m)} = [SP, GR, PORN, DEN, AT, RMLL, RLLD]^T , \qquad (2)$$

where GR - natural gamma ray [API], SP - spontaneous potential [mV], PORN - neutron porosity [per cent], DEN - density [g/cm^3], AT - acoustic travel time [μsec/m], RMLL – micro-laterolog [ohmm], RLLD - deep laterolog [ohmm].

The petrophysical relation between model parameters in Eq.(1) and the theoretical values of well-logging data detailed in Eq. (2) can be write in a general form as

$$\vec{d}^{(th)} = \vec{g}(\vec{m}, \vec{c}) , \qquad (3)$$

where $\vec{c}$ includes a great number of unvarying zone parameters and textural properties of rocks treated as constants through the inversion. In a point by point inversion approach means to known the value of zone and textural parameters in order to avoid ambiguity made by under determination (having much more unknowns than observations). Therefore, the j-th theoretical well-logging data can be computed by

$$d_j^{(th)} = g_j(m_1, \ldots, m_M),$$

where the data is only the function of M number of model parameters in the point. As it is seen in Eqs.(1) and (2), the point by point inverse problem is marginally over determined, thus the accuracy and the reliability of the estimation is relatively limited. Therefore, it is worth inverting data of a greater depth-interval jointly in one inversion procedure, which increase the over determination greatly. The so-called interval inversion method is based on the use of depth-dependent response functions that calculates the j-th theoretical well-logging data as

$$d_j^{(th)}(z) = g_k(m_1(z), m_2(z), \ \ldots \ , m_M(z)) . \qquad (4)$$

By an appropriate series expansion of model parameters gives us a chance to describe the vertical changes of model parameters in an arbitrary interval and determine petrophysical parameters together with also the layer-thicknesses in one inversion procedure (Dobróka et al. 2002, Szabó 2004). Because of the great extent of over determination interval inversion is more accurate and stable than the conventional point by point inversion methods.

3. Combined inversion method

A new interval inversion algorithm was developed that supports the determination of petrophysical parameters varying vertically in an arbitrary depth interval. Let us assume that the petrophysical model is consist of layer-wise homogeneous layers and one inhomogeneous one. The direct problem in Eq.(4) can be computed whereas the following discretization of the i-th model parameter in Eq.(1) can be made

$$m_i(z) = \sum_{q=1\,(q\neq q^*)}^{Q^{(i)}} m_q^{(i)}\left[u(z - Z_{q-1}) - u(z - Z_q)\right] + \sum_{l=1}^{L^{(i)}} B_l^{(i)}\left(z - Z_{q^*-1}\right)^{l-1} , \tag{5}$$

where m_q - characteristic parameter of the q-th layer (q^* - ordinal number of the inhomogeneous layer), B - series expansion coefficients (Q, L - number of series expansion coefficients need), and Z - layer-boundary coordinates, which all have to be determined by the interval inversion as the elements of the unknown model vector.

In practice the inverse problem is generally solved by local gradient-based linearized optimization methods that linearize response equations in Eq.(3). It leads to the solution of some sets of linear equations improving iteratively an initial model given in the near vicinity of the solution. It is quick enough and the estimation errors can also be computed, but the search can be trapped in a local optimum of the objective function with high probability. This problem is reformed by the global optimum searching methods, which are very effective techniques, but not so economic considering that they require long computational run time. To employ the advantages both optimization techniques we constructed a new algorithm, which is based on the successive combination of a float-encoded Genetic Algorithm (Michalewicz 1992) and the well-known Damped Least Squares method (Marquardt 1959). By this technique we start the optimization with GA, which perform random search in the parameter space independently from the initial model. Then after a definite number of iteration in the close vicinity of the global optimum we change for linearized optimization to accelerate the convergence and obtain parameter estimation errors.

Formulating the combined interval inversion algorithm the following objective function can be defined

$$\Phi = \left\{ \begin{array}{ll} \dfrac{\eta^2}{\displaystyle\sum_{p=1}^{P}\sum_{k=1}^{N}\left(\frac{d_{pk}^{(m)} - d_{pk}^{(th)}}{d_{pk}^{(m)}}\right)^2 + \eta^2} & , \quad if \;\; k < k^* \\ \displaystyle\sum_{p=1}^{P}\left(\sum_{k=1}^{N}\left(\frac{d_k^{(m)} - d_k^{(th)}}{d_k^{(m)}}\right)^2 + \varepsilon^2\sum_{i=1}^{M} m_i^2\right) & , \quad if \;\; k \geq k^* \end{array} \right\} , \tag{6}$$

where P - number of depth-points in the investigated interval, N - number of data in the point, k - the number of the actual iteration step, ε, η - damping factors. The k^* corresponds to a given iteration number, when the algorithm is switched over from global to linearized optimization. The first term in Φ is the fitness function of the actual model vector, which is estimated by the Genetic Algorithm. The other expression is the objective function of the Marquardt algorithm for the interval inversion problem.

The first step of the GA optimization phase of the combined interval inversion algorithm is the generation great number of 20-200 initial random models that according to the terminology of GA are called as individuals of a population. Each individual have a fitness value, which determines quantitatively the survival capability of the individual, whether it gets into a new generation of models or dies. In geophysics the fitness function is connected with an objective function - based on e.g. the distance between observed and calculated data -, which characterizes the goodness of the current geophysical model. During the GA process the fittest individuals are selected to the next generation. Consistently, in the last generation the individual with maximum fitness value (with minimal data distance) corresponds to the solution of the optimization problem. The first term of Eq.(6) is the fitness function of the interval inversion problem, which we have to maximize. To achieve the global maximum we applied three different kind of genetic operators as normalized geometric ranking selection, simple crossover and uniform mutation (Michalewicz 1992). In the $k=(k^*-1)$ iteration step the model with the maximal Φ fitness is accepted as an initial model for the linearized phase of the optimization algorithm.

From the $k=k^*$ iteration step Φ has to be minimized, which leads to the following solution for the vector of model parameters

$$\vec{m}^{(k)} = \vec{m}^{(k-1)} + \left(\underline{\underline{G}}^T \underline{\underline{G}} + \varepsilon^2 \underline{\underline{I}}\right)^{-1} \underline{\underline{G}}^T \left(\vec{d}^{(m)} - \underline{\underline{G}}\vec{m}^{(q-1)}\right) \qquad (k \geq k^*),$$

where $\underline{\underline{G}}$ denotes the Jacobi's matrix containing the first numerical derivatives of well-logging data with respect to model parameters in Eq.(3). If we have information about the covariance matrix in the data space, we can compute the parameter estimation errors by means of the square root of the diagonal elements of the covariance matrix of estimated model parameters (Menke 1984).

4. Numerical test on synthetic data

At first we consider it to be practical to test the combined interval inversion algorithm on synthetic well-logging data. We defined a petrophysical model of a four layered sedimentary sequence of strata that consists of shale and sandstone beds. MODEL-1 can be seen in Table I., where H denotes the layer-thicknesses and the rest of the parameters conform to the elements of model vector in Eq.(1). The model consists of three homogeneous layers and an inhomogeneous one, which is a hydrocarbon reservoir with relatively high porosity. In the third layer the petrophysical parameters were assumed to change vertically in depth by fourth-degree power functions as

$$\left.\begin{aligned} f_1(z) &= -0.3994z^4 + 1.6782z^3 - 2.3931z^2 + 1.2711z + 0.1014 \\ f_2(z) &= 0.4902z^4 - 2.2032z^3 + 3.6155z^2 - 2.3423z + 0.7735 \\ f_3(z) &= 1.3195z^4 - 5.7097z^3 + 8.1082z^2 - 3.9980z + 0.7379 \\ f_4(z) &= 1 - f_1(z) - f_3(z) \end{aligned}\right\}.$$

Table I.

H (m)	POR	SX0	SW	VSH	VSD
6.0	0.2	0.8	0.4	0.3	0.5
2.0	0.1	1.0	1.0	0.8	0.1
8.0	$f_1(z)$	0.8	$f_2(z)$	$f_3(z)$	$f_4(z)$
4.0	0.1	1.0	1.0	0.6	0.3

The theoretical well-logging data of MODEL-1 were calculated using response equations in Eq.(4). As input data to the inversion procedure we charged the synthetic data with 5 per cent Gaussian noise. This way the logs in Eq.(2) were created. For example the natural gamma ray log and the deep laterolog can be seen in Fig. 1. The unknowns of the interval inversion process came from Eq.(1) as layer characteristic parameters in layer 1, 2 and 4, and the additional B discretization coefficients from Eq.(4) in the third hydrocarbon-bearing layer.

An initial population of models with large average distance from the solution in the data and the model space was given. This problem failed to handle with independent linearized optimization that is why we applied a global optimization method. In Fig. 2., it can be seen that the local optima of objective function based on Eq.(6) can be avoided by using the GA algorithm in case of such a distant initial model. After $k^*=300$ iteration steps, in the close vicinity of the global optima, we changed for linearized optimization to accelerate the convergence. The estimated model proved to be very accurate as the relative data and model distances indicate also this

$$D_d = \sqrt{\frac{1}{PN}\sum_{p=1}^{P}\sum_{i=1}^{N}\left(\frac{d_{pi}^{(m)} - d_{pi}^{(th)}}{d_{pi}^{(m)}}\right)^2} = 5.16\ \%,$$

$$D_m = \sqrt{\frac{1}{PM}\sum_{p=1}^{P}\sum_{l=1}^{M}\left(\frac{m_{pl}^{(b)} - m_{pl}^{(e)}}{m_{pl}^{(m)}}\right)^2} = 1.09\ \%.$$

In the last iteration step applying linearized optimization based on Damped Least Squares method it was possible to give the estimation errors of the model parameters. The main diagonal elements of the computed covariance matrix corresponded to the model variances and after a square-root extraction the errors of the petrophysical parameters were computed. In Fig. 3. the petrophysical model and the estimated logs by interval inversion can be compared for porosity, shale and sandstone contents. A very good fitting can be seen in the figure, which is confirmed by the values of the model parameters with their confidence intervals in Table II. In case of homogeneous layer parameters the error is about in the range of 0.1 per cent and it still remains very small for the expansion coefficients in the third layer as well. Comparing the inversion results to Table I., it can be stated that interval inversion is very stable and able to give very accurate and reliable estimate for the unknown petrophysical parameters for all that the discretization coefficients have generally strong correlation among them. As for the computational run time, we experienced that a GA algorithm requires about 40-60 minutes to get an optimal solution, but the combined inversion method achieved it within 6 minutes.

Table II.

Layer	B	POR	SX0	SW	VSH	VSD
1	-	0.2003 (±0.0020)	0.8063 (±0.0046)	0.4002 (±0.0022)	0.2980 (±0.0029)	0.4978 (±0.0047)
2	-	0.0954 (±0.0037)	1.0000 (±0.0069)	1.0000 (±0.0095)	0.7998 (±0.0022)	0.1032 (±0.0080)
3	B_1	0.0991 (±0.0075)	0.8042 (±0.0034)	0.7608 (±0.0109)	0.7377 (±0.0079)	not inversion unknown
	B_2	1.2582 (±0.0530)		-2.2391 (±0.0629)	-3.9884 (±0.0552)	
	B_3	-2.3170 (±0.1142)		3.3857 (±0.1222)	8.0646 (±0.1195)	
	B_4	1.5956 (±0.0895)		-2.0142 (±0.0924)	-5.6695 (±0.0945)	
	B_5	-0.375 (±0.0229)		0.4397 (±0.0236)	1.3088 (±0.0244)	
4	-	0.0964 (±0.0024)	1.0000 (±0.0052)	1.0000 (±0.0066)	0.5981 (±0.003)	0.2987 (±0.0057)

4. Field case

After testing combined interval inversion method we applied it to the interpretation of real well-logging data. We inverted eight logs from a Hungarian hydrocarbon exploratory borehole. Focusing our attention to a 18m thick interval of WELL-1, we found two gas-bearing formations by cross-plot technique. These two gas-bearing sandstone formations are situated in the upper Pannonian (Pliocene) stage. By inversion it is possible to determine the petrophysical parameters in Eq.(1) so as to characterize the reservoir productivity quantitatively. In the interval inverse problem besides varying effective porosity conditions and unknown sand and shale contents we purposed to determine the movable and the irreducible hydrocarbon saturation as derived parameters from the estimated water saturations

$$SCHM(z) = SX0(z) - SW(z)$$
$$SCHR(z) = 1 - SX0(z).$$

In the zone investigated the following measurements were carried out in the well as natural gamma ray, spectral gamma ray (potassium, uranium and thorium separately), true resistivity, compensated density, compensated neutron porosity and acoustic travel time. The well logs to be inverted can be seen in Fig. 4. In the figure the disjunctions between compensated neutron and density logs indicate the reservoirs clearly.

At first we made a series expansion of layer-wise homogeneous petrophysical model beside unknown layer-thicknesses. A float-encoded GA was implemented for this problem, which determined the layer characteristic parameters and the boundary-coordinates at z=4.2m, 8.5m, 9.9m and 18.2m. In Fig. 5. the estimated saturation logs with the layer-boundary coordinates on the depth-scale can be found. Afterwards, taking advantage of the great extent of the over determined interval inversion problem we proposed to determine the vertical changes of petrophysical parameters within the lower interested gas-bearing layer. The petrophysical parameters were assumed to change in the fourth layer by four degree power functions in the series expansion. Consequently, the number of unknowns (petrophysical parameters and series expansion coefficients) increased slightly in comparison with the data available in the interval. We applied the previously detailed combined interval inversion method based on GA and linearized optimization. It improved a population of 100 models through 200 iterations. The maximal number of iteration steps was 300 (in case of individual GA inversion 3000 iteration steps were required to achieve the optimum). In Fig. 5. a higher resolution can be seen for the hydrocarbon saturation values for the fourth layer than it was obtained in case of layer-wise homogeneous petrophysical model. Approximately at depth 16 m, a transition zone between gas and connate water is shown, from below the movable gas saturation decreases versus depth. In Fig. 6. a significant difference in CPU run time can be seen between the global and the combined inversion method.

4. Conclusions

It was shown that the float-encoded GA inversion of well-logging data results in convergent solution estimating accurate and reliable petrophysical model parameters independently of the initial model. We started the algorithm with a great number of random petrophysical models and obtained always convergent solution. It turned out that global optimization methods have got some disadvantage as well. At first its convergence is too slow, on the other hand the computation of estimation errors can not be found out from only one inversion program run. We solved this problem by applying a combined inversion algorithm based on the joint use of global and linearized optimization, which is able to reduce the CPU time at least one order of magnitude and estimate the errors of petrophysical parameters in the last linearized iteration step.

In the paper we demonstrated two interval inversion techniques, which process well-logging data to determine the vertical change of the petrophysical parameters in one inversion procedure. At first homogeneous layer characteristic parameters were determined, and after inhomogeneous layer parameters were estimated in a more careful manner. It was shown that the latter technique is more suitable to characterize the reservoir rock for its hydrocarbon productivity with no failure in the quality of inversion.

5. Acknowledgements

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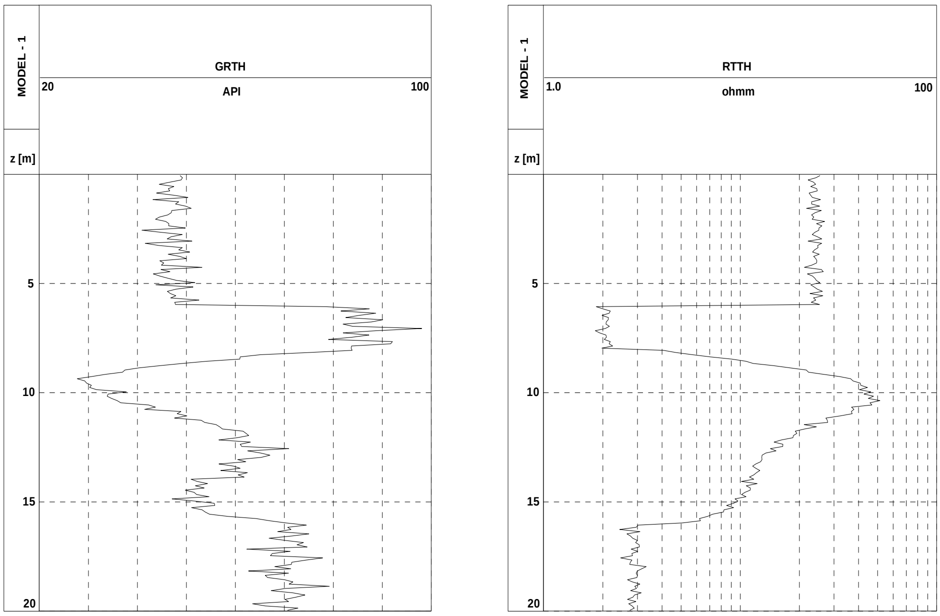

Fig. 1. Synthetic well-logging data computed from MODEL-1

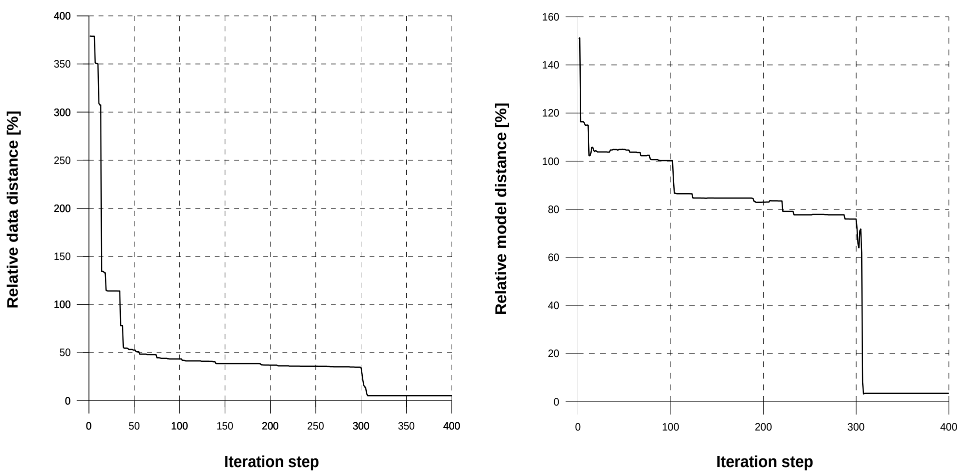

Fig. 2. Relative data- and model distance vs. iteration step

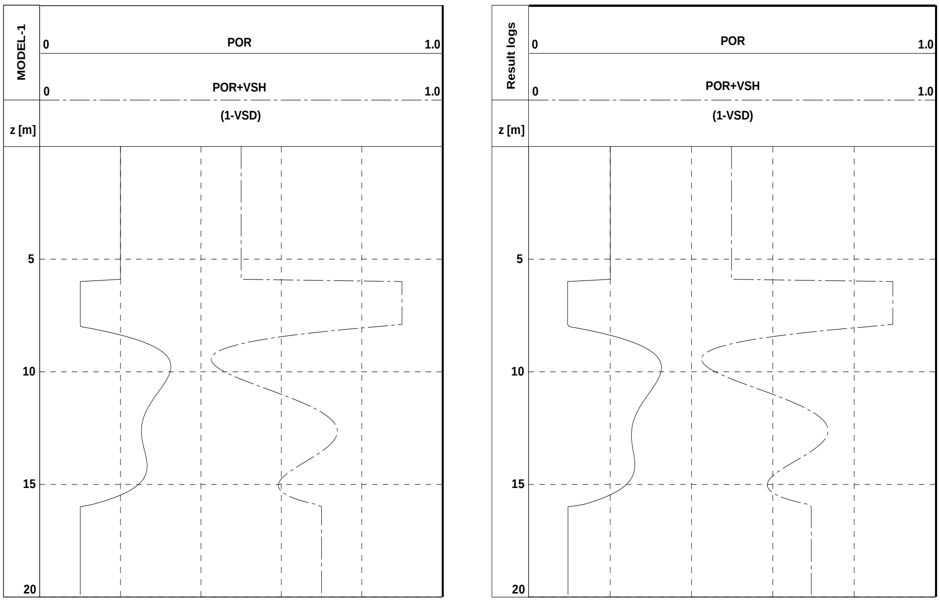

Fig. 3. MODEL-1 vs. interval inversion results

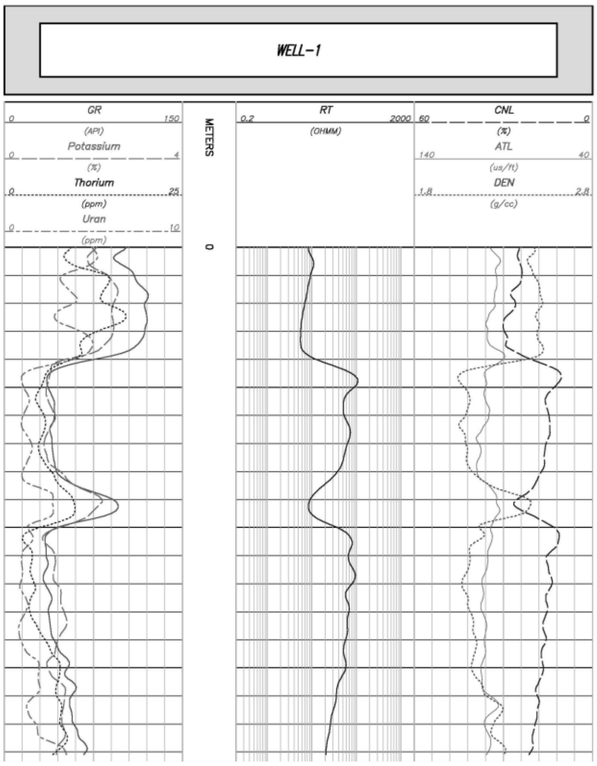

Fig. 4. Measured well logs from WELL-1

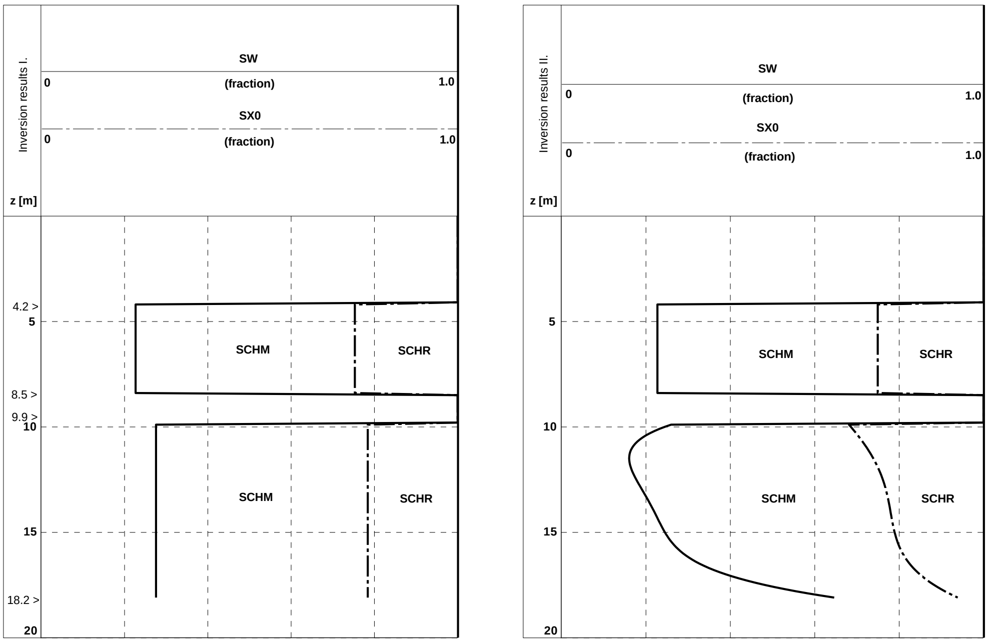

Fig. 5. Saturation logs estimated by interval inversion
(Inversion results I: unit step functions; II: power function in layer four)

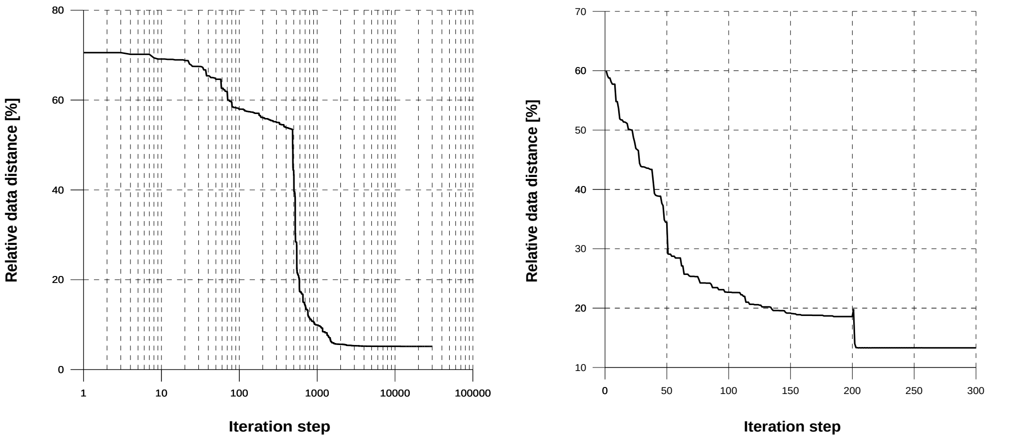

Fig. 6. Relative data distance vs. iteration step
(on the left - GA interval inversion, on the right - combined interval inversion)